Inspecting the Mechanism of Diagnostic Expectations: An Analytical Approach

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October 7, 2024

Abstract

In line with recent surge of behavioral economics, the diagnostic expectations (DE) paradigm has been adopted by many papers in macroeconomic and international literature: Under DE, new information influences expectations more strongly than under rational expectations. This paper contributes to a better understanding of diagnostic expectations by analytically solving some dynamic models. It starts with a canonical adjustment cost model to illustrate ways to deal with endogenous variables appearing in leads and lags. We also discuss a DE model with nominal rigidities—when prices are predetermined one period in advance—and show that the model builds in a moving-average behavior.

Keywords: Diagnostic Expectations, Rational Expectations.

JEL classification: E.

*Comments from Moto Shintani (discussant at 2024 JSIE conference), Seunghoon Na, and Donghoon Yoo are appreciated. The remaining errors are the author's.

1 Introduction

In line with recent surge of behavioral economics, the diagnostic expectations (DE) paradigm has been adopted by many papers in macroeconomic and international literature as reviewed in the ensuing paragraphs. Under DE, new information influences expectations more strongly than under rational expectations. However, in a dynamic setting, a few methods have been proposed as a way to incorporate DE. This paper contributes to a better understanding of DE by analytically applying these different methods in solving some dynamic models. It starts with a canonical adjustment cost model to illustrate ways to deal with endogenous variables appearing in leads and lags. We also (plan to) discuss a DE model with nominal rigidities—when prices are predetermined one period in advance—and show that the model builds in a moving-average behavior.

Having emerged as one important departure from rational expectations, diagnostic expectations (DE) have gained traction in the literature. Based on influential work on the representativeness heuristic by Kahneman and Tversky (1972), DE have been adapted in macroeconomic contexts in recent studies by Bordalo, Gennaioli, Ma, and Shleifer (2020), D'Arienzo (2020), Maxted (2020), Bordalo, Gennaioli, Shleifer, and Terry (2021), Bianchi, Ilut, and Saijo (2024), L'Huillier, Singh, and Yoo (2023), and Na and Yoo (2024), among others.[1] For a comprehensive review, see Gennaioli and Shleifer (2018) and Bordalo, Gennaioli, and Shleifer (2022).

Maxted (2020) and Bordalo, Gennaioli, Shleifer, and Terry (2021) incorporate DE in macro-finance frameworks. Specifically, Maxted (2020) shows that incorporation of DE into a macro-finance framework can reproduce several facts surrounding financial crises, whereas Bordalo et al. (2021) show that DE can quantitatively generate countercyclical credit spreads in a heterogeneous-firms model. D'Arienzo (2020) investigates the ability of DE to reconcile the overreaction of expectations of long rates relative to the expectations of short rates to news in bond markets. Ma, Ropele, Sraer, and Thesmar (2020) quantify the costs of managerial biases.

2 A Primer on Rational Expectations

Before delving into the discussion on diagnostic expectations, it is worthwhile to set up a simple model of adjustment costs under rational expectations. Even more, we start with a deterministic version of rational expectations: perfect foresight.

[1] Bianchi et al. (2024) and L'Huillier et al. (2023) propose a solution to solve a linear DSGE model under DE, with Bianchi et al. (2024) focusing on distant memory, where agents' reference distributions extend beyond one period. In an open-economy context, Na and Yoo (2024) introduce DE into small open economy (SOE) business cycle models to discuss macroeconomic fluctuation of the external balance.

2.1 Perfect Foresight

A deterministic version of adjustment cost model chooses the process of k_s for $s = t, t+1, t+2, ...$ to minimize the following objective function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]. \tag{1}$$

The first-order condition of this optimization problem (for a representative k_t) is

$$\begin{aligned} 0 &= k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta(k_{t+1} - k_t) \\ &= -\alpha\beta k_{t+1} + (1+\alpha+\alpha\beta)k_t - \alpha k_{t-1} - k_t^* \\ &= -\left[\alpha\beta L^{-1} - (1+\alpha+\alpha\beta) + \alpha L\right] k_t - k_t^*. \end{aligned}$$

Its solution is

$$k_t = \Lambda k_{t-1} + \alpha^{-1}\Lambda\sum_{s=t}^{\infty}\left[(\beta\Lambda)^{s-t} k_s^*\right], \tag{2}$$

where

$$\Lambda = \frac{(1+\alpha+\alpha\beta) - \sqrt{(1+\alpha+\alpha\beta)^2 - 4\alpha^2\beta}}{2\alpha\beta}. \tag{3}$$

As the degree of adjustment costs becomes bigger with α moving from zero to ∞, the degree of auto-regression gets bigger as well with Λ increasing from zero to unity.[2]

2.2 Rational Expectations

To introduce rational expectations in a simple fashion, let us assume that the target process is stochastic. Then our objective function is

$$\frac{1}{2}\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right], \tag{4}$$

which can—under the assumption of rational expectations—be expressed as follows:

$$\frac{1}{2}\left\{(k_t - k_t^*)^2 + \alpha(k_t - k_{t-1})^2 + \mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right]\right\}. \tag{5}$$

[2] As α goes to zero, so goes Λ to zero at the same speed. That is, $\alpha^{-1}\Lambda$ converges to unity such that k_t become k_t^*.

Under rational expectations, the first order condition is

$$0 = k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t\left[k_{t+1}\right] - k_t\right), \tag{6}$$

and its solution is

$$k_t = \Lambda k_{t-1} + (1-\Lambda)(1-\beta\Lambda)\mathbb{E}_t\left[\sum_{s=t}^{\infty}(\beta\Lambda)^{s-t}k_s^*\right]. \tag{7}$$

It is straightforward to show that

$$\alpha^{-1}\Lambda = (1-\Lambda)(1-\beta\Lambda) \tag{8}$$

based on the definition of Λ in (3). In the context of Calvo pricing, k_t and k_t^* are analogous to the price index and the contemporaneous marginal cost—with Λ being the probability of keeping the price of the previous period.

From now on—for the sake of notational simplicity—we assume that the exogenous target follows an autoregressive process,

$$k_t^* = \rho k_{t-1}^* + \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} N(0,1). \tag{9}$$

Under the AR(1) process, the rational expectations solution is reduced as follows:

$$k_t = \Lambda k_{t-1} + \Gamma k_t^*, \tag{10}$$

where

$$\Gamma = \frac{\Lambda}{\alpha\left(1-\beta\Lambda\rho\right)} = \frac{(1-\Lambda)(1-\beta\Lambda)}{(1-\beta\Lambda\rho)} = \frac{1}{1+\alpha\beta\left(\beta^{-1}+1-\Lambda-\rho\right)}. \tag{11}$$

In the case of random walk ($\rho = 1$), dynamics would follow $k_t = \Lambda k_{t-1} + (1-\Lambda)k_t^*$.

3 Three Versions of Diagnostic Expectations

The key equation capturing the essence of diagnostic expectations modifies rational expectations as follows:

$$\mathbb{E}_t^\theta\left[X_t\right] = \mathbb{E}_t\left[X_t\right] + \theta\left(\mathbb{E}_t\left[X_t\right] - \mathbb{E}_{t-J}\left[X_t\right]\right), \tag{12}$$

for $J = 1, 2, 3, ...$ In this paper, we focus on the case of $J = 1$.

In applying diagnostic expectations to a dynamic problem such as adjustment costs, three versions have been considered. This paper—to differentiate them from one another—applies three adjectives: careful, casual and cavalier. The former two versions start from two objective functions that are slightly from each other while being equivalent under rational expectations. The last version—instead of basing itself on an objective function—modifies the first order condition from rational expectations.

3.1 The Careful

After considering how to apply diagnostic expectations to a dynamic optimization setting with due care, BIS and LSY set the diagnostic expectations version of an objective function as specified in (5):

$$\frac{1}{2}\left\{(k_t-k_t^*)^2+\alpha(k_t-k_{t-1})^2+\mathbb{E}_t^\theta\left[\sum_{s=t+1}^{\infty}\beta^{s-t}\left[(k_s-k_s^*)^2+\alpha(k_s-k_{s-1})^2\right]\right]\right\}. \tag{13}$$

Noting that k_t appears three times in this objective function—twice as it is and once in the diagnostic expectations operator—careful derivation of the first order condition yields the following optimality condition:

$$0=k_t-k_t^*+\alpha(k_t-k_{t-1})-\alpha\beta\mathbb{E}_t^\theta\left[k_{t+1}-k_t\right]. \tag{14}$$

The solution of this difference equation is

$$k_t=\Lambda k_{t-1}+\Gamma k_t^*+\Phi^{Careful}\varepsilon_t, \tag{15}$$

where

$$\Phi^{Careful}=\frac{\theta\alpha\beta\Gamma(\Lambda+\rho-1)}{1+\alpha(1+\beta)-\alpha\beta(\Lambda+\Lambda\theta-\theta)}=\frac{\alpha\beta\Gamma(\Lambda+\rho-1)\theta}{1+\alpha+\alpha\beta(1-\Lambda)(1+\theta)}. \tag{16}$$

It is interesting to note that the Φ-term—the additional dependence on the innovation—disappears as the model becomes static (i.e. $\alpha\to 0$.) This is true to the latter cases as well. A way to resuscitate diagnostic expectations in this limiting ($\alpha\to 0$) case is to assume that diagnostic expectations are applied to the first k_t^* term.

Another interesting property of $\Phi^{Careful}$ is that it is an increasing function of ρ. In particular, $\Phi^{Careful}$ takes the value of zero, when ρ is equal to $1-\Lambda$.

3.2 The Casual (or The Offhanded)

Some papers (e.g. Bounader and Elekdag, 2024) are also not very explicit about stating optimization problem, which may allow for an objective function analogous to (4):

$$\frac{1}{2}\mathbb{E}_t^{\theta}\left[\sum_{s=t}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right], \tag{17}$$

whose first order condition would be

$$0 = \mathbb{E}_t^{\theta}\left[k_t - k_t^*\right] + \alpha(\mathbb{E}_t^{\theta}\left[k_t\right] - k_{t-1}) - \alpha\beta\mathbb{E}_t^{\theta}\left[k_{t+1} - k_t\right]. \tag{18}$$

Its solution is

$$k_t = \Lambda k_{t-1} + \Gamma k_t^* + \Phi^{casual}\varepsilon_t \tag{19}$$

where

$$\Phi^{casual} = \frac{\left[1 - \Gamma - \alpha\Gamma + \alpha\beta\Gamma(\Lambda + \rho - 1)\right]\theta}{\left[1 + \alpha + \alpha\beta(1 - \Lambda)\right](1 + \theta)} = 0. \tag{20}$$

That is, this solution is equivalent to that of rational expectations–at least in our model of adjustment costs. The casual version does not yield any overreaction or underreaction.

3.3 The Cavalier

Instead of starting from an optimizing problem, one could simply (or offhandedly) replace the DE operator for the RE operator in (6) as follows:

$$0 = k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t^{\theta}\left[k_{t+1}\right] - k_t\right). \tag{21}$$

The solution of this difference equation is

$$k_t = \Lambda k_{t-1} + \Gamma k_t^* + \Phi^{Cavalier}\varepsilon_t, \tag{22}$$

where [TBC]

$$\Phi^{Cavalier} = \frac{\theta\alpha\beta\Gamma(\Lambda + \rho)}{1 + \alpha(1 + \beta) - \alpha\beta(\Lambda + \Lambda\theta)} = \frac{\alpha\beta\Gamma(\Lambda + \rho)\theta}{1 + \alpha + \alpha\beta(1 - \Lambda - \Lambda\theta)} \tag{23}$$

[TBC] Unlike the two previous cases, it is the denominator of $\Phi^{Cavalier}$—rather than its nominator—that could be zero as parameters vary. Specifically, bifurcation happens as the value of θ increases.

4 Conclusion

Do we choose by theory checking on our prior or by empirics using expectations data?

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5 Appendix: Comparing Three DE Solutions under a Dynamic Constraint

The adjustment cost model is simple, but most dynamic models involve a dynamic budget constraint such as bond accumulation. This appendix posits two cases for different timing conventions of a stock variable, and in each case, three DE solutions are compared against each other.

5.1 Case I. The budget constraint given by B_{t+1} and B_t

In this case, the stock variable is measured at the beginning of the period.

5.1.1 Careful:

The preferences of the representative household are given by the utility function:

$$\log C_t + \chi \log(1 - N_t) + \mathbb{E}_t^\theta \sum_{s=t+1}^{\infty} \left[\beta^{s-t} \left(\log C_s + \chi \log(1 - N_s) \right) \right] \tag{24}$$

where $\mathbb{E}_t^\theta[\cdot]$ denotes the diagnostic expectation operator conditional on information available at time t. The household maximizes it subject to the following constraint:

$$C_t + B_{t+1} = w_t N_t + R_t B_t + \Pi_t \tag{25}$$

The FOCs for this *Careful* representation are $\frac{1}{C_t} = \Lambda_t$ and $\Lambda_t = \beta \mathbb{E}_t^\theta [R_{t+1} \Lambda_{t+1}]$. Thus, we have

$$\frac{1}{C_t} = \beta \mathbb{E}_t^\theta \left[\frac{R_{t+1}}{C_{t+1}} \right] \tag{26}$$

from which with log-linearization, we get <u>the consumption Euler equation (careful)</u>

$$\hat{c}_t = \mathbb{E}_t^\theta [\hat{c}_{t+1} - \hat{r}_{t+1}] \tag{27}$$

5.1.2 Casual:

Modify the utility function as follows:

$$\mathbb{E}_t^{\theta} \sum_{s=t}^{\infty} \left[\beta^{s-t} \left(\log C_s + \chi \log(1 - N_s) \right) \right] \tag{28}$$

Thus, FOCs for this *Casual* representation are $\mathbb{E}_t^{\theta}\left[\frac{1}{C_t}\right] = \mathbb{E}_t^{\theta}[\Lambda_t]$ and $\mathbb{E}_t^{\theta}[\Lambda_t] = \beta\mathbb{E}_t^{\theta}[R_{t+1}\Lambda_{t+1}]$. With log-linearization, we get $\mathbb{E}_t^{\theta}[\hat{\lambda}_t] = -\mathbb{E}_t^{\theta}[\hat{c}_t]$ and $\mathbb{E}_t^{\theta}[\hat{\lambda}_t] = \mathbb{E}_t^{\theta}[\hat{r}_{t+1} + \hat{\lambda}_{t+1}]$.
Thus, the consumption Euler equation (casual) is given by

$$\mathbb{E}_t^{\theta}[\hat{c}_t] = \mathbb{E}_t^{\theta}[\hat{c}_{t+1} - \hat{r}_{t+1}]. \tag{29}$$

5.1.3 Cavalier:

We can easily show that the equilibrium condition for its counterpart RE model is:

$$\frac{1}{C_t} = \beta\mathbb{E}_t\left[\frac{R_{t+1}}{C_{t+1}}\right] \tag{30}$$

With log-linearization, we have

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1} - \hat{r}_{t+1}] \tag{31}$$

Thus, the consumption Euler equation (cavalier) is given by

$$\hat{c}_t = \mathbb{E}_t^{\theta}[\hat{c}_{t+1} - \hat{r}_{t+1}]. \tag{32}$$

In this case, the cavalier version happens to be equivalent to the careful version.

5.2 Case II. The budget constraint given by B_t and B_{t-1}

The stock varable can instead be measured at the end of the period.

5.2.1 Careful:

The same utility function as before but the budget constraint is given by:[3]

$$C_t + B_t = w_t N_t + R_{t-1} B_{t-1} + \Pi_t \tag{33}$$

The FOCs are $\frac{1}{C_t} = \Lambda_t$ and $\Lambda_t = \beta \mathbb{E}_t^\theta [R_t \Lambda_{t+1}]$. Thus, we have

$$\frac{1}{C_t} = \beta \mathbb{E}_t^\theta \left[\frac{R_t}{C_{t+1}} \right] \tag{34}$$

from which with log-linearization, we get the consumption Euler equation (careful)

$$\hat{c}_t = \mathbb{E}_t^\theta [\hat{c}_{t+1} - \hat{r}_t] \tag{35}$$

5.2.2 Casual:

We again modify the utility function as follows:

$$\mathbb{E}_t^\theta \sum_{s=t}^{\infty} \left[\beta^{s-t} \left(\log C_s + \chi \log(1 - N_s) \right) \right] \tag{36}$$

Thus, FOCs for this *Casual* representation are $\mathbb{E}_t^\theta \left[\frac{1}{C_t} \right] = \mathbb{E}_t^\theta [\Lambda_t]$ and $\mathbb{E}_t^\theta [\Lambda_t] = \beta \mathbb{E}_t^\theta [R_t \Lambda_{t+1}]$.
With log-linearization, we get $\mathbb{E}_t^\theta [\hat{\lambda}_t] = -\mathbb{E}_t^\theta [\hat{c}_t]$ and $\mathbb{E}_t^\theta [\hat{\lambda}_t] = \mathbb{E}_t^\theta [\hat{r}_t + \hat{\lambda}_{t+1}]$.
Thus, the consumption Euler equation (casual) is given by

$$\mathbb{E}_t^\theta [\hat{c}_t] = \mathbb{E}_t^\theta [\hat{c}_{t+1} - \hat{r}_t] \tag{37}$$

5.2.3 Cavalier:

We can easily show that the equilibrium condition for its counterpart RE model is:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{R_t}{C_{t+1}} \right] \tag{38}$$

[3] An equivalent representation of this contraint is $A_{t+1} = R_t \left(A_t + w_t N_t + \Pi_t - C_t \right)$ by defining $A_t = R_{t-1} B_{t-1}$. If we timed the rate of return in this budget constraint—and the budget constraint in the main text—as the latest timing of the stock variable, the three versions of the FOC's in this case would be equivalent to those from Case I.

With log-linearization, we have

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \hat{r}_t \tag{39}$$

Thus, <u>the consumption Euler equation (cavalier)</u> is given by

$$\hat{c}_t = \mathbb{E}_t^{\theta}[\hat{c}_{t+1}] - \hat{r}_t. \tag{40}$$

In Case II, all three versions yield different different solution.