A Search and Matching Model of Heterogeneous Wage Rigidity and Missing Trickle Down*

Kazuhiro Teramoto†

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Abstract

I propose a search-and-matching model in which sluggish wage cyclicality and volatile unemployment endogenously arise. The model features job-ladder and vacancy-chain effects arising from on-the-job search and replacement hiring. In equilibrium, employed job searches are procyclical, and thereby, the quality of job seekers increases during expansions. As a result, employers have increased opportunities to replace the existing poorly-matched employees with well-qualified workers, and wages are less responsive especially for poorly-matched/lower-paying jobs due to the enhanced firm's outside option value in wage negotiations. In the calibrated model, wage growth in response to aggregate productivity shocks in the lower income group is roughly half that in the upper income group, and aggregate productivity shocks account for about three-fourths of observed volatility in unemployment rate.

Key Words: Search-and-matching models, On-the-job search, Replacement hiring, Wage rigidity, Business cycles

JEL Classification: E24, E32, J31, J63, J64

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†Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, Japan. 186-8603. Email: k.teramoto@r.hit-u.ac.jp

1 Introduction

Wage rigidities have been attracting a great deal of interest in the macro-labor literature, particularly in their role in increasing the cyclical volatility of labor market variables (Shimer, 2005a; Hall, 2005, and the following literature on the unemployment volatility puzzle). In measuring wage cyclicality, a number of studies (e.g., Pissarides, 2009; Rudanko, 2009; Haefke, Sonntag, and Van Rens, 2013; Gertler, Huckfeldt, and Trigari, 2020) have emphasized the distinction between new hires and existing workers, building on the implications from the canonical Diamond-Mortensen-Pissarides (DMP) model featuring that the job creation is more sensitive to wages of new hires. However, recent evidence from administrative data suggests that wage cyclicality differs considerably across income groups. Specifically, Guvenen, Ozkan, and Song (2014) report that, in the boom and bust years around 2007, workers in the top 10% of labor earnings and above experienced a change in labor-earnings growth of about 15 percentage points, while those in the middle of labor earnings experienced a change of only about 5 percentage points. In sum, the evidence suggests cyclicality of wages is week particularly for middle earners. Despite the considerable attention devoted to wage cyclicality in the labor-search literature, yet little has been studied on the causes of the difference in cyclicality across income groups and its roles to cyclical labor market behavior.

In this study, I propose a novel search-and-matching model that explains sluggish cyclical behavior of wages for middle earners and show that the model generates a enhanced cyclical volatility of unemployment. Putting differently, I provide a search-and-matching model in which sluggish cyclical behavior of wages and enhanced cyclical volatility of labor market flows *endogenously* arise. The model is a variant of the DMP search-and-matching model that is designed to explain the salient features of the US labor market dynamics: procyclical upgrading of job match quality (Barlevy, 2002; Gertler, Huckfeldt, and Trigari, 2020)[1] and vacancy posting for replacement hiring (Mercan and Schoefer, 2020; Elsby, Michaels, and

[1] The fact that job-to-job flows are highly procyclical in the US labor market has been confirmed from a variety of data sources. Using the Current Population Survey (CPS), Fallick and Fleischman (2004) find that the monthly employer-to-employer (E-to-E) flows are procyclical, and Mukoyama (2014) reconstructs the time-aggregation-adjusted measure and reconfirms the procyclicality. Nagypál (2008) uses the longitudinal data set from the Survey of Income and Program Participation (SIPP) to observe E-to-E transitions at weekly frequency and finds that E-to-E transitions account for 49 percent of separations on average and they are procyclical. Bjelland, Fallick, Haltiwanger, and McEntarfer (2011) and Hyatt and McEntarfer (2012a) use national job-to-job flow statistics derived from the Longitudinal Employer-Household Dynamics (LEHD) infrastructure files and find that E-to-E transitions sharply fell in 2001 recession and the Great Recession.

Ratner, 2020).

The model presented in this paper, in its basic model structure, follows the canonical DMP model, such as Pissarides (1985) and Shimer (2005a). Workers and firms, who are all risk-neutral and homogeneous a priori, are bilaterally matched as a result of a random search. The key distinction is that job flows (job creation and destruction) and worker flows (hires and separations) are distinguished due to the following features. First, it considers variation in job match quality and allows workers to conduct a costly on-the-job search for a better-matched job. According to the extent to which a job change leads to wage increases, employed workers optimize the intensity of job search, which allows for cyclical variations in quits. Second, it considers long-lived jobs arising from a sunk fixed cost and a less than infinitely elastic job-creation process, as in Fujita and Ramey (2007), making the model depart from the free entry of vacancies. With this feature, jobs endure even after the match ends due to the worker's quit. Third, it allows for replacement hiring, as in Mercan and Schoefer (2020). Namely, firms are allowed to re-post a vacancy to replace positions vacated by quits. Thus, vacancies are not only driven by the creation of new jobs, but also by quits. Putting together, a worker can quit the existing match to upgrade job match quality, and the vacancy induced by the quit becomes an employment opportunity for other workers.

Wages are decided in a flexible Nash bargaining. Nevertheless, the aforementioned features affect the outcomes of the wage bargaining. In contrast to the canonical DMP model assuming free entry of vacancy postings, the value of a job vacancy, which is the outside option value to firms in wage negotiations, can vary over time. In the model, the value of a job vacancy is tied with the composition of job seekers in the labor market. Specifically, an increase in the number of employed job seekers relative to unemployed job seekers raises the probability of hiring a new well-qualified worker and increases the value of a job vacancy. With free entry of vacancy postings, this labor-supply composition effect only affects the size of vacancy postings; On the other hand, with the inelastic job-creation process, it affects not only the size of vacancy but also wages of the existing employees since the outside option value to firms is no longer zero.[2]

[2]The equilibrium wages in this model have another feature distinguishing them from those in the canonical DMP model, albeit its quantitative implications prove to be small. It is efficiency wage against worker turnover, that is, firms face the trade-off that lowering their wages increases instantaneous profits, but also increases quits and costs of replacing workers, resulting in paying efficiency wages to discourage incumbent workers from quitting. Early studies by Salop (1979), Stiglitz (1974), and Schlicht (1978) have applied the

In order to access the cyclical behavior of the model, the above framework is integrated into the real business cycle (RBC) model with extensive labor adjustment of Merz (1995) and Andolfatto (1996). I show that introducing both endogenous on-the-job search and replacement hiring enhances the response of unemployment and reduces the responses of wages to productivity shocks. A positive productivity shock increases the net return to job creation, thereby increasing new job creations on impact. In response to increased vacancies for new jobs, employed job searches become more intensive. As is well known, in the random-search environment, procyclical employed searches have an ambiguous overall effect on unemployment volatility: on the one hand, increased employed searches benefit unemployed workers by stimulating job creation further; on the other hand, increased employed searches increase labor market congestion and crowd out unemployed workers (Pissarides, 1994; Nagypál, 2006; Martin and Pierrard, 2014; Moscarini and Postel-Vinay, 2018). The presence of replacement hiring alleviates the crowding-out effect because, even if a vacancy is matched to an employed worker, another vacancy to replace the quitter is posted, and this process continuously occurs until a vacancy is matched to an unemployed worker. Through this vacancy chains effect (Akerlof, Rose, and Yellen, 1988), the negative effect of increased employed job searches on unemployed workers are mitigated over time. Besides, increased quits-driven vacancies encourage on-the-job search further, which increases quits and quits-driven vacancies further. In this manner, the presence of replacement hiring has amplifying effects on job-to-job flows as well as unemployment-to-employment flows.

In the presence of inelastic entry to job creation, procyclical searches by employed workers makes firms pushy in wage negotiations because increased employed searches raise chances for firms to replace their existing poorly-matched employees with well-qualified workers. This force dampens the response of wages particularly for incumbent workers in low-paying jobs to productivity shocks. The sluggish cyclicality wages for incumbent workers in low-paying jobs spills over into wages for new hires. In wage negotiations, a job changer with multiple employment opportunities (new and former jobs) views working at the former job, rather than becoming unemployed, as an outside option. Thus, she results in bargaining over the

principal-agent theories to show that, in the presence of costs of replacing workers, employers offer a higher wage than the market-clearing wage to discourage workers from quitting. I formulate such trade-off within a wage bargaining framework and show it results in lower bargaining power for firms and higher equilibrium wages, offering the conditions for the deterministic bargaining problems to work well.

difference in wages between new and former jobs.

The model is calibrated to match long-run features of the US labor market for the quantitative evaluation. In my calibration, the combination of endogenous on-the-job search and replacement employment more than doubles the cyclical volatility of unemployment, vacancies, and job-finding probability, whereas it reduces the cyclical volatility of the average wage per worker by 30 percent. As a result, the calibrated model is able to produce large cyclical volatility of the labor market quantities and relatively low volatility of the average wage, comparable to the US data. In the model, not only total employment but also employment share of better-matched/high-paying employment are highly procyclical, in line with evidence documented by Haltiwanger, Hyatt, Kahn, and McEntarfer (2018) and Moscarini and Postel-Vinay (2008, 2012). Even though this cyclical change in worker's composition contributes to increasing the cyclical volatility of the average wage, very sluggish behaviors of wages paid to each job lead to observed rigidity in the average wage, consistent with evidence provided by Gertler, Huckfeldt, and Trigari (2020). Further quantitative analyses show that procyclicality of the value of a vacancy is crucial in producing the sluggish behavior of wages. In particular, I show that, although an increase in the possibility of replacement hiring and an increase in the elasticity of entry to job creation both enhance the cyclical volatility of unemployment, only the former leads to wage rigidity; The former increases procyclicality of the value of a vacancy, while the latter decreases it.

Contributions to Literature This paper makes several contributions to the macro-labor literature. First, this study proposes a resolution to the unemployment volatility puzzle. Following seminal works by Shimer (2005a) and Hall (2005), a large number of studies have appealed for the introduction of some forms of wage rigidity in search-and-matching models (e.g., Krause and Lubik, 2007; Arseneau and Chugh, 2008; Costain and Reiter, 2008; Hall and Milgrom, 2008; Gertler and Trigari, 2009; Christiano, Eichenbaum, and Trabandt, 2016, 2021). As Pissarides (2009) pointed out that it is wage rigidity of new hires that plays a central role in generating the amplifying effects, the distinction between new hires and exiting employees has been emphasized in measuring wage cyclicality. Among them, the closest study to mine is Gertler, Huckfeldt, and Trigari (2020) who offer a variant of the DMP model with cyclical job changes involving upgrading of job match quality and wage rigidity of new hires. Several recent studies (Bils, Chang, and Kim, 2022; Schoefer, 2021; Carlsson and Westermark, 2022) provide the generalized DMP models to show that wage rigidity of incumbent workers also plays an important role in amplifying cyclical volatility of labor markets. This study differs from them in two important aspects. First, in contrast to the existing studies cited above, which have focused on the heterogeneity of wage cyclicality between new hires and existing employees, this study draws attention to the heterogeneity across different wage levels. Second, whereas many of the above-cited studies examine how exogenously-imposed wage rigidities affect the cyclical volatility of labor markets, this study provides a search-and-matching model in which both sluggish cyclical behavior of wages and large cyclical volatility of labor markets arise endogenously.

The second contribution to the macro-labor literature is to offer a new microfounded mechanism for sluggish cyclical fluctuations in wages. Within the labor-search literature, Rudanko (2009) considers a directed search model in which risk-neutral firms post optimal long-term contracts featuring wage smoothing to attract risk-averse workers, and shows that the cyclical volatility of unemployment does not increase as long as wages of new workers are highly procyclical. A recent paper Fukui (2020) further enriches the scope of Rudanko's (2009) analysis by considering on-the-job search as in the wage-posting model of Burdett and Mortensen (1998), and shows that the equilibrium contract leads to wage rigidity for new hires as well as incumbent workers. Whereas, Menzio and Moen (2010) consider a directed search model that allows for replace hiring and show that the optimal wage policy prescribes a downward rigid wage for new hires, which increases the unemployment volatility. Within the DMP search-and-matching framework, Morales-Jiménez (2022) shows that the delay in wage responses due to informational frictions enhances the unemployment volatility. Compared with these studies, this study is novel in focusing on cyclical fluctuations in the outside option value to employers in wage setting as a source of wage rigidity and showing that it has heterogeneous impacts across workers.

This study also contributes to the literature on replacement hiring. An extensive empirical literature has documented the sizable presence of quits-driven hires based on a variety of establishment-level data (e.g., Faberman and Nagypál, 2008; Davis, Faberman, and Haltiwanger, 2012; Lazear and Spletzer, 2012; Elsby, Michaels, and Ratner, 2020). In response to them, the recent search-and-matching models literature has shown a growing interest in the role of replacement hiring in labor market dynamics. Among many, the closest studies are Mercan and Schoefer (2020) and Elsby, Michaels, and Ratner (2020), who explicitly model replacement hiring and emphasize that vacancy chains arising from replacement hiring can have amplification effect on cyclical fluctuations of employment. This study extends the theoretical literature by merging cyclical on-the-job search and investigates the roles played by replacement hiring in the cyclical behavior of wages as well as labor market quantities.

The Structure of the Paper The rest of this article is organized as follows. Section 2 lays out the model. Section 3 derives the equilibrium wages. Section 4 discusses the calibration strategy and presents quantitative results. Section 5 offers concluding remarks.

2 Model

The model builds on the work of Merz (1995) and Andolfatto (1996), who integrated the DMP search-and-matching framework into the RBC model. It is distinguished from the Merz-Andolfatto model in the following three respects: (i) long-lived jobs arising from fixed entry costs, (ii) variation in job match quality and job-to-job transitions through on-the-job search, and (iii) possibility of replacement hiring. Putting them together, employed workers can quit their existing job to move to another job, while the firms recruit new workers to replace the quitters. Thus, there are worker flows (employment and turnover) not involving job flows (job creation and destruction), and vice versa.

2.1 Model Environment

Time is discrete. Let $t = 0, 1, 2, \dots$ index time. At the beginning of each period, an aggregate labor productivity shock ε_t is realized. Then, given the renewed wage contract. workers in an existing employment relationship (incumbent workers) choose how intensively they conduct on-the-job searches. Then, job seekers and job vacancies are matched in the labor market, and wage contracts for newly matched workers are made. Then, based on the reallocated workforce, final goods are produced. At the end of the period, exogenous job separation and destruction occur.

The economy consists of an infinite mass of infinitely-lived firms and a representative household consisting of a unit mass of infinitely-lived workers. The representative household provides perfect insurance against idiosyncratic risk on employment state within the family members, as in Merz (1995) and Andolfatto (1996). The household also holds the ownership of firms. Firms and workers are risk neutral and atomistic, each maximizing the expected present value of current and future payoffs discounted by the stochastic discount factor of the representative household. Let $\Lambda_{t,t+\tau}$ denote the stochastic discount factor from time t to $t+\tau$ for $\tau \geq 1$.

Each firm can hold at most one job at a time. A job is a productive opportunity that produces $\hat{y}_t > 0$ units of final good when it is matched to a particular worker. The labor productivity is determined by the product of the two components: $\hat{y}_t = y_t\epsilon$. The first component y_t is aggregate labor productivity that follows the exogenous stochastic process:

$$\log(y_t) = \rho_y \log(y_{t-1}) + (1-\rho_y)\log(y) + \sigma_y \varepsilon_t, \qquad \varepsilon_t \sim N(0,1) \tag{1}$$

with $|\rho_y| < 1$, $y = 1$, and $\sigma_y > 0$. The second component ϵ represents job match quality, assumed to take two values: $\epsilon \in \{\epsilon_L, \epsilon_H\}$ with $\epsilon_L = 1$ and $\epsilon_H > 1$. As will be detailed below, job match quality depends on whether the job is matched to an inexperienced worker, i.e., unemployed job seeker, or to an experienced worker, i.e., employed job seeker.

Workers are either employed or unemployed. Let u_t denote mass of workers unemployed *at the beginning* of each period t. Unemployed workers collect the flow benefit $b \in (0, y)$ from nonworking and look for a job with a constant intensity normalized to one. Unemployed search is costless. If matched to a job, they begin working in the subsequent period. With this timing assumption, once a worker becomes unemployed, she remains unemployed for at least one period.

Employed workers with mass $\ell_t = 1 - u_t$ hold an existing match to a particular job. Let $\ell_{L,t} \geq 0$ and $\ell_{H,t} \geq 0$ denote the mass of employed workers poorly and well matched to their current job, respectively. By construction, it holds that $\ell_{L,t} + \ell_{H,t} = \ell_t$. Employed workers are allowed to search for a new job and choose intensity $e \geq 0$ of on-the-job search. It is assumed that conducting on-the-job search with search incurs $\widehat{\mathcal{S}}(e) = \mathbb{1}_{\{e>0\}}S + \mathcal{S}(e)$, denominated in final goods. The first term $S > 0$ denotes the fixed cost of conducting on-the-job searches and $\mathcal{S}(e) \geq 0$ is variable costs satisfying $\mathcal{S}(0) = 0$, $\mathcal{S}'(e) > 0$, and $\mathcal{S}''(e) > 0$.

Thus, when not searching ($e = 0$), no cost is incurred: $\widehat{\mathcal{S}}(0) = 0$. I assume that the fixed cost exists but it is very small so that only poorly-matched workers conduct on-the-job search, i.e., $e_{H,t} = 0$ and $e_{L,t} \geq 0$, in equilibrium.[3] The variable cost function is assumed $\mathcal{S}'(0) = 0$ and $\lim_{e\to\infty} \mathcal{S}'(e) = \infty$, ensuring a unique interior optimum for $e_{L,t}$. In what follow, I denote the efficiency weighted mass of job searchers in time-t labor market by $s_t = u_t + e_{L,t}\ell_{L,t}$.

The one-to-one match structure implies that, at the beginning of each period t, there exists mass ℓ_t of firms (or jobs) matched to a particular worker. Each period, there are infinitely many potential entrants, who must incur a one-time sunk cost of $k_t > 0$ to create a long-lived job. It is assumed that the job-creation cost increases in the size of entry at a time $v_{n,t} \geq 0$, so $k_t = k(v_{n,t}) > 0$ with $k'(v_{n,t}) \geq 0$.[4] Jobs are assumed to be vacant at the creation and continue to survive, either being filled or vacant, until exogenously destructed, as in Fujita and Ramey (2007). With these structures, job vacancies are made up of vacancies for newly created jobs and those for jobs that were created in the past but is currently vacant. The above described job-creation cost allows for imperfect substitution between job vacancies for new and existing jobs.

A firm with a vacant job is allowed to post a job vacancy by bearing the flow cost $\zeta_v > 0$. A job vacancy is randomly matched to either an unemployed or employed job seeker. After matched to a worker, the firm incurs a post-match hiring cost $\varphi_H > 0$ to integrate the new employee into the workforce.[5] The job match quality depends on whether it is matched to an unemployed or employed worker. I assume that it is better when matched to an employed job seeker than an unemployed worker, which motivates employed workers to engage in a costly on-the-job search for a better-matched job. I adopt the standard timing assumption that, if an employed job seeker is matched to a new job, she starts working at the new firm immediately (without waiting until the subsequent period).

[3]This is line with the survey evidence in Faberman, Mueller, Şahin, and Topa (2022) that on-the-job search is more intense for workers in low-paying jobs.

[4]This job-creation cost function follows Fujita and Ramey's (2007) specification and exhibits the feature that creating an additional job becomes more costly the larger the size of the job created at one time. This captures the nature of entry that adjustment costs and scarcity of profitable opportunities increase due to increased entry (Fujita and Ramey, 2007, Footnote 11 and Section 6). Coles and Moghaddasi Kelishomi (2018) provides a microfoundation for the job-creation process by extending the modeling strategy in Diamond (1982).

[5]Recent works (e.g, Pissarides, 2009; Ljungqvist and Sargent, 2017) has focused on the role of the post-match hiring cost in amplifying responses of labor market quantities to productivity shocks in the DMP labor-search models.

At the end of each period, three types of idiosyncratic shocks occur to firm-worker pairs. First, a better match quality deteriorates with Poisson probability p_{HL}. In this case, the match continues but labor productivity will be down to $y\epsilon_L$ in the subsequent period. Second, a matched pair is exogenously separated with Poisson probability σ, resulting in that a randomly-chosen fraction σ of employed workers leaves the match and becomes unemployed. Note that, when hit by the match-separation shock, the job can survive while it becomes vacant. Finally, a job is destructed with Poisson probability δ, regardless of whether it is filled or vacant.

As a consequence, a filled job becomes vacant due to quit or exogenous match-separation shock. If the job becomes vacant, the firm holding it is able to start recruiting a worker in the subsequent period (as long as the job has not been destroyed). In order to examine the role played by replacement hiring, it is assumed that the firm can re-post a job vacancy for replacement hiring with probability $\gamma \in [0, 1]$. An extreme case with $\gamma = 0$ means that the job is effectively destructed immediately upon separations, as in the canonical DMP model.

The wage is decided in a bargaining period by period. The firm-worker pairs in existing matches negotiate the continuation wages immediately after observing the aggregate productivity shock ε_t. The continuation wages differ depending on their match quality, denoted by $w_{oH,t}$ and $w_{oL,t}$. The wage for job changers $w_{e,t}$ is set right after the match is formed.

The timing of the events within a period is summarized as follows:

1. The productivity shock ε_t is realized.
2. The firm-worker pairs in existing matches negotiate the continuation wage.
3. New jobs are created by the entrants.
4. Employed workers choose intensity of on-the-job search. Vacant firms choose whether to post a job vacancy. In the labor market, the mass m_t of matches are formed.
5. The wages for job changers are negotiated.
6. The final goods are produced and wages are paid according to the contract.
7. A better match deteriorates with probability p_{HL}.
8. A match-separation shock arrives with probability σ.

9. A job-destruction shock arrives with probability δ.

2.2 Job and Worker Flows

Each period, (initially vacant) new long-lived jobs are created under free entry. The free-entry condition drives the net value of creating a job N_t to zero:

$$N_t = V_t - k(v_{n,t}) = 0, \tag{2}$$

where V_t represents the expected present value of the current and future profits earned by a firm posting a job vacancy in time t. The equilibrium condition (2) ensures a positive net value of posting a job vacancy: $V_t = k(v_{n,t}) > 0$, implying that all the firms with a vacant job are willing to post a job vacancy.[6] The mass of vacancies in time t is $v_t = v_{o,t} + v_{n,t}$, where $v_{o,t}$ is the mass of jobs created in the past but being vacant at the beginning of period t.

Each period, a labor market opens, in which job vacancies and job seekers randomly meet. Given the mass of job vacancies v_t and the (efficiency weighted) mass of job searchers s_t, a mass of hiring per period $m_t > 0$ is determined by the constant-return-to-scale (CRS) matching function: $m_t = M(v_t, s_t)$, where M is increasing and concave in both its arguments. The CRS matching function implies the probability with which a vacancy is matched to a job seeker $m_t/v_t = q(\theta_t)$, where $\theta_t = v_t/s_t$ denotes the labor market tightness, and the probability with which a unit of search activity leads to a match $m_t/s_t = f(\theta_t)$. As standard in the literature, it is assumed that $q(\theta)$ satisfies $q'(\theta) < 0$, $q(0) = 1$, and $\lim_{\theta\to\infty} q(\theta) = 0$ and $f(\theta)$ satisfies $f'(\theta) > 0$, $f(0) = 0$, and $\lim_{\theta\to\infty} f(\theta) = 1$. The random search process implies that a job vacancy is matched to an unemployed worker with probability $(u_t/s_t)q(\theta_t)$ and it is matched to an employed worker with probability $(1 - u_t/s_t)q(\theta_t)$. Similarly, an unemployed worker is matched to a job with probability $f(\theta_t)$ and a poorly matched worker changes her job with probability $e_{L,t}f(\theta_t)$.

With the timing assumptions made above, there are three ways in which an employed worker leaves the existing match: (i) an exogenous job destruction, (ii) an exogenous match

[6] More precisely, let $\tilde{V}_t$ denote the value for a firm with a vacant job at the beginning of time t. It is given by $\tilde{V}_t = \max\{V_t, 0\}$, where V_t is the value the firm would obtain if it posts a vacancy and 0 is the value if it is inactive. The free entry condition ensures that $\tilde{V}_t = V_t$.

separation, and (iii) a job change. In the first and second cases, he worker becomes unemployed in the subsequent period. In the third case, the worker remains employed but works at a different firm. For the firm side, the firm loses its job in the first case, while it holds a vacant job (with probability γ) and can posts a vacancy for the job in the subsequent period.

As a result of the labor-market reallocation, the mass of workforce at each match quality level is determined as $\hat{\ell}_{H,t} = \ell_{H,t} + e_{L,t}f(\theta_t)\ell_{L,t}$ and $\hat{\ell}_{L,t} = (1 - e_{L,t}f(\theta_t))\ell_{L,t}$. Recall that better job match quality deteriorates with probability p_{HL} at the end of each period. The implied laws of motion for employment and unemployment stocks are respectively given by

$$\ell_{H,t+1} = (1-\sigma)(1-\delta)(1-p_{HL})\hat{\ell}_{H,t},$$

$$\ell_{L,t+1} = (1-\sigma)(1-\delta)[\hat{\ell}_{L,t} + p_{HL}\hat{\ell}_{H,t} + f(\theta_t)u_t],$$

and

$$u_{t+1} = (1-\tilde{f}(\theta_t))u_t + [\sigma + (1-\sigma)\delta]\,\ell_t, \tag{3}$$

where $\tilde{f}(\theta_t) = (1-\sigma)(1-\delta)f(\theta_t)$ is the probability with which an unemployed worker at the beginning of period t becomes employed at the beginning of the subsequent period.

Vacancy is also a stock variable. The law of motion for the vacancy stock is given by

$$v_t = (1-\delta)\left[1-(1-\sigma)q\left(\theta_{t-1}\right)\right]v_{t-1} + v_{in,t}, \tag{4}$$

where the first term represents the mass of surviving vacancies carried over from the previous period and the second term is the *inflow* to the vacancy stock between time $t-1$ and t. It is given by

$$v_{in,t} = v_{n,t} + \gamma(1-\delta)\left[e_{L,t-1}f(\theta_{t-1})\ell_{L,t-1} + \sigma\{\ell_{t-1} - e_{L,t-1}f(\theta_{t-1})\ell_{L,t-1}\}\right],$$

where the first term is the mass of jobs newly created in time t and the second term the mass of vacancies posted for replacement hiring. Recall that replacement hiring is possible with probability γ. In the square bracket, $e_{L,t-1}f(\theta_{t-1})\ell_{L,t-1}$ is the mass of jobs vacated due to quits, and $\sigma\{\ell_{t-1} - e_{L,t-1}f(\theta_{t-1})\ell_{L,t-1}\}$ is the mass of jobs vacated due to the exogenous separation shock.

2.3 Value Functions

I setup the value functions for each agent. In what follow, $\mathbb{E}_t$ denotes the expectation operator conditional on information available at time t.[7]

Workers At the beginning of each period, workers are in either of the three employment statuses: unemployed; employed and poorly matched to the job; employed and well matched to the job. Let $E_{u,t}$, $E_{oL,t}(w_{oL,t})$, and $E_{oH,t}(w_{oH,t})$ be the expected present values of current and future payoffs for a worker in respective employment statuses.

With the notations introduced above, the value for the unemployed worker is given by

$$E_{u,t} = b + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{u,t+1}\right] + \tilde{f}(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{u,t+1}\right)\right].$$

A worker holds the existing match chooses intensity of on-the-job search, taking the contract wage at the current job given. As described in Section 2.1, only poorly-matched workers are willing to conduct on-the-job search. Thus, $E_{oL,t}(w_{oL,t})$ is formulated as follows:

$$E_{oL,t}(w_{oL,t}) = \max_{e_L \geq 0}\left\{-\mathcal{S}(e_L) + e_L f(\theta_t)H_{e,t}(w_{e,t}) + (1 - e_L f(\theta_t))C_{L,t}(w_{oL,t})\right\},$$

where $H_{e,t}(w_{e,t})$ is the value she gains upon a job change, which is given by

$$\begin{aligned} H_{e,t}(w_{e,t}) =& w_{e,t} + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oH,t+1}\right] + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oH,t+1}\right)\right] \\ &+ p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{oH,t+1}\right)\right], \end{aligned}$$

while $C_{o,t}(w_{oL,t})$ is the value of staying at the existing match:

$$C_{L,t}(w_{oL,t}) = w_{oL,t} + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oL,t+1}\right] + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oL,t+1}\right)\right].$$

The optimal search intensity $e_{L,t}(w_{oL,t})$ satisfies

$$\mathcal{S}'(e_{L,t}(w_{oL,t})) = f(\theta_t)\left(H_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t})\right) \quad \text{if} \quad H_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t}) > 0 \tag{5}$$

and $e_{L,t}(w_{oL,t}) = 0$ otherwise. When (5) holds, $e_{L,t}(w_{oL,t})$ is decreasing in $w_{oL,t}$, i.e.,

[7] The productivity shock ε_t is realized at the beginning of each period, and thus, when making a decision at each period t, all the model agents are informed of the realizations up to t, $\{\varepsilon_s\}_{s=0}^t$.

$e'_{L,t}(w_{oL,t}) = -f(\theta_t)/\mathcal{S}''(e_{L,t}(w_{oL,t})) < 0$, implying that employed search is more intensive when the wage at the current job is lower. The envelop theorem shows the worker's marginal value from increasing the continuation wage is $E'_{oL,t}(w_{oL,t}) = 1 - e_{L,t}(w_{oL,t})f(\theta_t) \in (0, 1]$.

Well-matched workers stay at the current match. Thus, $E_{oH,t}(w_{oH,t})$ is given by

$$\begin{aligned} E_{oH,t}(w_{oH,t}) =& w_{oH,t} + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oH,t+1}\right] + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oH,t+1}\right)\right] \\ &+ p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{oH,t+1}\right)\right]. \end{aligned}$$

Firms Let $J_{oL,t}(w_{oL,t})$ and $J_{oH,t}(w_{oH,t})$ be the value of a firm that is poorly matched and well matched to a worker at the beginning of period t, respectively. Using free-entry condition (2), they are respectively given by

$$\begin{aligned} J_{oL,t}(w_{oL,t}) =& \left[1 - e_{L,t}(w_{oL,t})f(\theta_t)\right]\left(y_t\epsilon_L - w_{oL,t}\right) \\ &+ (1-\delta)\left(\begin{array}{l} \mathbb{E}_t\left[\Lambda_{t,t+1}J_{oL,t+1}\right] \\ +\left[\sigma + (1-\sigma)e_{L,t}(w_{oL,t})f(\theta_t)\right]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\gamma V_{t+1} - J_{oL,t+1}\right)\right] \end{array}\right), \end{aligned} \tag{6}$$

and

$$J_{oH,t}(w_{oH,t}) = y_t\epsilon_H - w_{oH,t} + (1-\delta)\left(\begin{array}{l} \mathbb{E}_t\left[\Lambda_{t,t+1}J_{oH,t+1}\right] + \sigma\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\gamma V_{t+1} - J_{oH,t+1}\right)\right] \\ +(1-\sigma)p_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(J_{oL,t+1} - J_{oH,t+1}\right)\right] \end{array}\right).$$

Note that, in (6), firms are aware of how the worker chooses search intensity according to the continuation wage. Given $e'_{L,t}(w_{oL,t}) < 0$, firms face the trade-off that lowering their wages increases the instantaneous profit, but it also increases quits and costs of replacing workers.

The value for a vacant job is given by

$$V_t = -\zeta_v + q(\theta_t)\left[(u_t/s_t)M_{u,t} + (1 - u_t/s_t)M_{e,t}(w_{e,t}) - \varphi_H\right] + (1 - q(\theta_t))\hat{V}_t, \tag{7}$$

where $\hat{V}_t = (1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]$ represents the value for a job that remains vacant when the time-t labor market closes. In (7), $M_{u,t}$ and $M_{e,t}(w_{e,t})$ represent the values a job has after the post-match hiring cost φ_H is sunk if it is matched to an unemployed worker and an employed job seeker, respectively. They are given by

$$M_{u,t} = (1-\delta)\left(\sigma\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\gamma V_{t+1} - J_{oL,t+1}\right)\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}J_{oL,t+1}\right]\right),$$

and

$$M_{e,t}(w_{e,t}) = y_t\epsilon_H - w_{e,t} + (1-\delta)\left(\begin{array}{l} \mathbb{E}_t\left[\Lambda_{t,t+1}J_{oH,t+1}\right] + \sigma\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\gamma V_{t+1} - J_{oH,t+1}\right)\right] \\ +(1-\sigma)p_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(J_{oL,t+1} - J_{oH,t+1}\right)\right] \end{array}\right).$$

Note that the composition of job seekers (unemployed versus employed job seekers) affects V_t. If $M_{e,t}(w_{e,t}) > M_{u,t}$, which is the case in the steady state in my calibration, an increase in the share of employed job seekers $(1 - u_t/s_t)$ enhances the value for a vacant job.

2.4 Wage Bargaining

The matched firm-worker pair sets the contract wage according to the Nash bargaining solution. As commonly assumed in the literature, firms do not have commitment on the future wage payments, so all wages are renegotiated period by period.

Wages for workers in existing employment relationships The continuation wages are negotiated just after observing the realization of the productivity shock at each period. Once wage contracts are made, they are never renegotiated within the period.[8]

I begin with the bargaining problem for well-matched firm-worker pairs. Since the worker do not search on the job, the joint surplus from the match $S_{oH} \equiv (J_{oH,t}(w_{oH,t}) - V_t) + (E_{oH,t}(w_{oH,t}) - E_{u,t})$ is independent of the wage. Thus, maximizing the Nash product $(J_{oH,t}(w_{oH,t}) - V_t)^{1-\eta}(E_{oH,t}(w_{oH,t}) - E_{u,t})^{\eta}$ with $\eta \in [0,1)$ being the worker's bargaining weight gives the surplus sharing rule:

$$E_{oH,t}(w_{oH,t}) - E_{u,t} = \eta S_{oH}. \tag{8}$$

On the other hand, the bargaining problem for poorly matched firm-worker pairs is more complicated because of voluntary quits. I assume that, although the contract wage cannot be contingent on the worker's search intensity, firms are aware of how the wage affects the

[8] This bargaining scheme rules out the possibility that workers who received an outside offer will exploit it to raise their wage at the current job, as in the sequential auctions protocol of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006). However, Mortensen (2005, page 99) notes that counteroffers to outside offers are not common in reality: "Unlike in the market for academic economists in the United States, making counteroffers is not the norm in many labor markets. More typically, a worker who informs his employer of a more lucrative outside option is first congratulated and then asked to clear out immediately." He proposes one of the reasons why counteroffers are unusual is that counteroffer strategies encourage employees to seek outside employment opportunities, increasing future wage burdens and costs of replacing workers.

worker's search intensity and quit rate. Taking this into account, the joint surplus from the match $S_{oL,t} \equiv (J_{oL,t}(w_{oL,t}) - V_t) + (E_{oL,t}(w_{oL,t}) - E_{u,t})$ depends on the wage payment, explicitly written as $S_{oL,t} = S_{oL,t}(w_{oL,t})$.[9] This may make feasible payoff set non-convex, and if this is the case, the deterministic Nash bargaining solution is not applicable because a lottery over different wage contracts is preferred (Shimer, 2006).

Proposition 1. *If (i) $\mathcal{S}'''(e) \leq 0$ for all $e \geq 0$, (ii) $0 \leq w_{oL,t} \leq y_t \epsilon_L$, and (iii) $J_{oL,t}(w_{oL,t})$ is strictly decreasing in $w_{oL,t}$, the bargaining set is convex.*

Proof of Proposition 1. See Appendix A. □

In Section 4, I will show the conditions (i), (ii), and (iii) are satisfied the standard parameterization and thus the convexity of the bargaining set is naturally ensured around the steady state.[10] Under the convex bargaining set, maximizing the Nash product $(J_{oL,t}(w_{oL,t}) - V_t)^{1-\eta}(E_{oL,t}(w_{oL,t}) - E_{u,t})^{\eta}$ results in

$$E_{oL,t}(w_{oL,t}) - E_{u,t} = r_t(w_{oL,t}) S_{oL,t}(w_{oL,t}), \tag{9}$$

where $r_{o,t}(w_{o,t})$ represents the worker's effective bargaining power given by

$$r_t(w_{oL,t}) = \frac{\eta}{\eta + (1-\eta)\mu_t(w_{oL,t})/\epsilon_t(w_{oL,t})}.$$

Proposition 2. *When the worker is conducting on-the-job search, i.e., $e_L > 0$, the worker's effective bargaining power is greater than η: $r_t(w_{oL,t}) > \eta$.*

Proof. It is enough to show that $\mu_t(w_{o,t}) < \epsilon_t(w_{o,t})$. See Appendix A for expressions for $\mu_t(w_{o,t})$ and $\epsilon_t(w_{o,t})$. □

Proposition 2 means that the worker increases the bargaining power, taking advantage of the option to quit. This leads to an upward pressure on the wage, as in the models of efficiency wage against worker turnover (Salop, 1979; Stiglitz, 1974; Schlicht, 1978).

[9] More specifically, the wage is no longer perfectly transferable between the firm and the worker because the worker's marginal gain from increase in wage payment is higher than the firm's marginal loss from increase in wage payment: $\epsilon_t(w_{o,t}) > \mu_t(w_{o,t})$.

[10] There is no guarantee of convexity out of steady-state. However, as the departure from convexity is not substantial, considering small transactions costs of the wage lottery can avoid stochastic wage contracts (Gertler and Trigari, 2009, Section 2.4).

Wages for job changers Job changers negotiate their wages under the assumption that they would be recalled to their former employer if they were unsuccessful in matching with the new employer.[11] Thus, they view the value of working at the former job $C_{L,t}(w_{oL,t})$, instead of the value of unemployment, as the fallback value. Thus, maximizing the Nash product $(J_{e,t}(w_{e,t}) - \hat{V}_t)^{1-\eta}(E_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t}))^{\eta}$ results in

$$H_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t}) = \eta S_{e,t} \tag{10}$$

where $S_{e,t} \equiv (J_{e,t}(w_{e,t}) - \hat{V}_t) + (E_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t}))$ is the joint surplus from the match. Hence, job changers exploit multiple job opportunities and receive a higher wage than the wage in the form job: $w_{e,t} > w_{oL,t}$.

2.5 Representative Household

Within the member of the representative household, there is a perfect risk sharing regarding employment opportunities. Thus, their labor incomes are pooled and equally consumed to maximize the expected present value of current and future utilities from consumption discounted by the factor $\beta \in (0, 1)$:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s u(c_{t+s}) \right],$$

subject to the household's budget constraint: $c_t + d_{t+1} \leq (1 + i_t)d_t + \bar{w}_t \ell_t + bu_t + \Pi_t$, where c_t is each worker's final-good consumption in time t, d_t is holding of one-period risk-free bonds maturing in time t, i_t is interest rate of the bond, $\bar{w}_t$ is the average wage of employed workers, and Π_t include the net lump-sum transfer to the household and the firms' profits. The period utility function $u(\cdot)$ satisfies $u'(\cdot) > 0$, $u''(\cdot) < 0$ and constant relative risk aversion $\phi = cu''(c)/u'(c) > 0$. The intertemporal consumption-and-saving problem of the household implies the stochastic discount factor given by $\Lambda_{t,t+1} = \beta u'(c_{t+1})/u'(c_t)$.

[11] With the timing assumptions made above, the wage of new hires from unemployment is $w_{oL,t}$. This is in line with the evidence provided by GHT that new hires wages from unemployment are no more cyclical than for existing workers.

2.6 Resource Constraints

The gross output of final good in each period t is $Y_t \equiv y_t(\epsilon_L \hat{\ell}_{L,t} + \epsilon_H \hat{\ell}_{H,t})$. It is used for consumption, costs of conducting on-the-job search, and hiring costs:

$$Y_t = c_t + \sum_{j \in \{L,H\}} \hat{\mathcal{S}}(e_{j,t})\ell_{j,t} + v_{n,t}k(v_{n,t}) + v_t\zeta_v + q(\theta_t)v_t\varphi_H.$$

The net supply of risk-free bond is zero: $d_t = 0$.

3 Equilibrium Wage Dynamics

In this section, I derive how wages are set in the model. Solving (8) gives[12]

$$\begin{aligned} w_{oH,t} =& (1-\eta)\left[b + (\tilde{f}(\theta_t) - \tilde{p}_{HL})\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{u,t+1}\right)\right]\right] \\ &+ \eta\left[y_t\epsilon_H + \tilde{p}_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(J_{oL,t+1} - V_{t+1}\right)\right]\right] \\ &- \eta\left[V_t - (1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right], \end{aligned} \tag{11}$$

with $\tilde{p}_{HL} = (1-\sigma)(1-\delta)p_{HL}$. In (11), the first line in the right-hand side stands for the contribution of the worker's surplus, while the sum of the second and third lines stand for the contribution of of the firm's surplus. Note that, unlike the canonical DMP model assuming free entry of vacancies, the terms in the third line are not constant—an increase in the value of vacancy, that is the firm's outside option in the wage bargaining, places downward pressure on $w_{oH,t}$.

Solving (9) gives

$$\begin{aligned} &[1 - e_{L,t}f(\theta_t)]w_{oL,t} \\ &\quad = (1 - r(w_{oL,t}))\left[b + \mathcal{S}(e_{L,t}) - e_{L,t}\mathcal{S}'(e_{L,t}) + \tilde{f}(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{u,t+1}\right)\right]\right] \\ &\quad + r(w_{oL,t})\left[\left[1 - e_{L,t}f(\theta_t)\right]y_t\epsilon_L - e_{L,t}\tilde{f}(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(J_{oL,t+1} - V_{t+1}\right)\right]\right] \\ &\quad - r(w_{oL,t})\left[V_t - (1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right], \end{aligned} \tag{12}$$

where the sum of the first and second lines of the right-hand side expresses the contribution of the worker's surplus, while the sum of the third and fourth lines expresses the contribution of

[12] See Online Appendix B for the derivation.

of the firm's surplus. Similarly to (11), an increase in the value of vacancy V_t decreases $w_{oL,t}$. In comparison with (11), there are two differences. First, in (12), the resultant worker's share of the total surplus is $r(w_{oL,t})$, rather than η. Second, an increase in the expected return of on-the-job search $\mathcal{S}'(e_{L,t}) = f(\theta_t)\left(H_{e,t}(w_{e,t}) - C_{L,t}\right)$ have a negative impact on $w_{oL,t}$.

Finally, solving (10) gives

$$\begin{aligned} w_{e,t} =& (1-\eta)\left[w_{oL,t} + (1-\sigma)(1-\delta)(1-p_{HL})\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{u,t+1}\right)\right]\right] \\ &+ \eta\left[y_t\epsilon_H + \tilde{p}_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(J_{oL,t+1} - V_{t+1}\right)\right]\right]. \end{aligned} \tag{13}$$

In (13), the value of the worker's outside option is the wage at the former job $w_{oL,t}$, instead of b, and thus, the wage of job changers is affected by that of incumbent workers.

Given the wage of each job, the average wage per (employed) worker is given by

$$\bar{w}_t = \frac{w_{oH,t}\ell_{H,t} + w_{oL,t}(1 - e_{L,t}f(\theta_t))\ell_{L,t} + w_{e,t}e_{L,t}f(\theta_t)\ell_{L,t}}{\hat{\ell}_t}.$$

In addition, the average wage of incumbent workers is given by

$$\bar{w}_t^{inc} = \frac{w_{oH,t}\ell_{H,t} + w_{oL,t}(1 - e_{L,t}f(\theta_t))(\ell_{L,t} - \tilde{f}(\theta_{t-1})u_{t-1})}{\ell_{H,t} + (1 - e_{L,t}f(\theta_t))(\ell_{L,t} - \tilde{f}(\theta_{t-1})u_{t-1})},$$

and the average wage of new hires is given by

$$\bar{w}_t^{new} = \frac{w_{e,t}e_{L,t}f(\theta_t)\ell_{L,t} + w_{oL,t}(1 - e_{L,t}f(\theta_t))\tilde{f}(\theta_{t-1})u_{t-1}}{e_{L,t}f(\theta_t)\ell_{L,t} + (1 - e_{L,t}f(\theta_t))\tilde{f}(\theta_{t-1})u_{t-1}}.$$

4 Quantitative Analysis

In this section, I use the model to quantitatively evaluate how well the model is capable to capture cyclical labor market dynamics in the United States. Following the literature, this study focuses on the first-order response to shocks to the aggregate labor productivity that follows the stationary stochastic process (1).

4.1 Functional Specification

For the quantitative analysis, I specify functional forms of the on-the-job search cost function $\mathcal{S}(e_{L,t})$, the matching function $M(v_t, s_t)$, and the job-creation cost function $k(v_{n,t})$.

The on-the-job search cost function takes the following power form: $\mathcal{S}(e_{L,t}) = S_0(e_{L,t}/e_L)^{1+1/\xi}$ with $S_0 = \frac{\varsigma}{1+1/\xi}$ where $\varsigma > 0$ is a constant to scale the cost and $\xi > 0$ is the inverse elasticity of the marginal search cost with respect to the search intensity, and $e_L > 0$ being the steady-state value of $e_{L,t}$.

Regarding the matching function, I follow den Haan, Ramey, and Watson (2000) and adopt $M(v,s) = vs/(v^\psi + s^\psi)^{1/\psi}$ with $\psi > 0$. The implied job-finding and vacancy-filling probabilities are $f(\theta) = \theta/(1+\theta^\psi)^{1/\psi} \in [0,1]$ and $q(\theta) = 1/(1+\theta^\psi)^{1/\psi} \in [0,1]$, respectively.

As in Coles and Moghaddasi Kelishomi (2018), the job-creation function takes the following isoelastic form: $k(v_{n,t}) = a_k(v_{n,t}/v_n)^{1/\kappa}$ with $a_k > 0$ being the steady-state value of the job-creation cost, κ being the elasticity of new jobs with respective to the value of a vacancy (see (2)), and $v_n > 0$ being the steady-state value of $v_{n,t}$. Applying implicit function theorem to free-entry condition (2) gives the following steady-state condition:

$$-\frac{\partial v_n}{\partial v} = \frac{v_n \kappa}{a_k}\left[\frac{(1-\beta(1-\delta))a_k + \zeta_z}{q(\theta)}\right]\left(\frac{\theta}{q(\theta)}\right)^{\psi-1}\frac{1}{s} > 0, \tag{14}$$

showing that, holding others constant, crowding out of job creation due to an increase in replacement hiring increases in κ/a_k. Namely, κ/a_k governs the degree of rivalry between new job creations and replacement hiring. In an extreme case with $\kappa/a_k = \infty$, all vacancies are perfectly substitute, and thus, the vacancy stock v_t is determined so that $V_t = a_k$ for all t as in the canonical DMP model that assumes free entry of vacancies.

4.2 Parameterization

The model is parameterized so that it accounts for the long-run US labor market dynamics. Table 1 lists values assigned to the model parameters.

One period of the model is normalized to one month. The top panel of Table 1 lists the values of the parameters chosen from external sources. The following four parameters are common to the RBC literature: the subjective discount factor is set $\beta = (0.99)^{1/3}$ so that the implied annual real interest rate is 4 percent; the relative risk aversion is set $\phi = 1.0$ (logarithmic utility); the autoregressive parameter in the labor productivity process ρ_y and standard deviation of innovations in productivity process σ_y are chosen to target the first-order autocorrelation and standard deviation of the HP-filtered real per capita output

Table 1: Parameter Values

Description		Value
Parameters externally calibrated		
β	Monthly discount factor	$(0.990)^{1/3}$
ϕ	Degree of relative risk aversion	1.0
ρ_y	Persistence of productivity process	$(0.970)^{1/3}$
σ_y	Standard deviation of innovations in productivity process	0.0065
η	Worker's bargaining power	0.50
κ	Job-creation elasticity	0.265
Parameters internally calibrated		
ψ	Matching elasticity parameter	1.385
σ	Match-separation probability	0.010
δ	Job-destruction probability	0.024
p_{HL}	Probability of job-match-quality deterioration	0.011
ς	Scale of on-the-job search costs	0.062
ξ	Elasticity of on-the-job search intensity	1.700
a_k	Steady-state job-creation cost	0.101
ζ_v	Vacancy-posting cost	0.168
φ_H	Post-match hiring cost	0.535
b	Flow value of nonworking	0.703
ϵ_H/ϵ_L	Job-match-quality improvement on a job change	1.075

($\rho_y = (0.97)^{1/3}$ and $\sigma_y = 0.0065$). The worker's bargaining power is set $\eta = 0.5$, a value commonly used in the search-and-matching literature. The elasticity of job creation with respective to the value of a vacancy is set $\kappa = 0.265$, a Coles and Moghaddasi Kelishomi's (2018) estimate. Note that, as shown in (14), the degree of substitution between vacancies for new jobs and exiting jobs is determined by κ/a_k, and thus, given the calibration strategy described below, the choice of a value for κ has little effect on quantitative implications. Finally, the availability of replacement hiring is assumed to be $\gamma = 1$ (full availability) for the benchmark case. To examine the role of replacement hiring in business cycles, I simulate the case with $\gamma = 0$ (see Section 4.4 in detail).

I then calibrate 11 parameters, (i) the matching elasticity parameter ψ; (ii) the match-separation probability σ; (iii) the job-destruction probability δ; (iv) the probability of deterioration in the quality of job match p_{HL}; (v) the scaling parameter in on-the-job search cost function ς; (vi) the elasticity of on-the-job search intensity with respect to the expected return ξ; (vii) the average job-creation cost a_k; (viii) the vacancy-posting cost ζ_v; (ix) the

Table 2: Calibration Targets

Target	Corresponding model variable	Value
Monthly UE probability	$\tilde{f}(\theta) = (1-\sigma)(1-\delta)f(\theta)$	45%
Vacancy-filling probability	$\tilde{q}(\theta) = (1-\sigma)(1-\delta)q(\theta)$	71%
Unemployment rate	$u = (\sigma + (1-\sigma)\delta)/(\sigma + (1-\sigma)\delta + \tilde{f}(\theta))$	7.0%
Probability of wage reduction within a job	$(p_{HL}\ell_H)/[(1-e_L(w_{oL})f(\theta))\ell_L + \ell_H]$	0.6%
Quit rate	$e_L(w_{oL})f(\theta)\ell_L/\ell$	2.5%
Share of on-the-job search cost to output	$\mathcal{S}(e_L(w_{oL}))/Y$	4.0%
Vacancy crowding out	$-\partial v_n/\partial v$	0.1183
Share of re-posted vacancies in total job openings	$(v_{in} - v_n)/v_{in}$	56%
Share of post-match hiring cost in total hiring costs	$q(\theta)\varphi_H/(\zeta_v + q(\theta)\varphi_H)$	70%
Average income replacement ratio	$b/\bar{w}$	71%
Average annual earning growth on job change	$\bar{w}_e/w_{oL}$ with $\bar{w}_e = (w_e + 11 \times w_{oH})/12$	9.0%

post-match hiring cost φ_H; (x) the flow value from nonworking b; (xi) the degree of job match quality improvement on a job change ϵ_H/ϵ_L to match the long-run moments of the US labor market dynamics. Specifically, I target the following empirical moments, (i) the average monthly transition probability from unemployment to employment of 45 percent (Shimer, 2005a); (ii) the average vacancy-filling probability of 71 percent (den Haan, Ramey, and Watson, 2000); (iii) the average unemployment rate of 7.0 percent; (iv) the average probability of wage reduction for job stayers is 0.6 percent (Grigsby, Hurst, and Yildirmaz, 2021);[13] (v) the average monthly quit rate of 2.5 percent (Fujita and Nakajima, 2016); (vi) the share of costs of on-the-job search to output of 4 percent; (vii) the share of re-posted vacancies in total vacancy inflow of 56 percent (Mercan and Schoefer, 2020); (viii) the crowding out of job creation due to a vacancy for existing job is 0.1183, (Mercan and Schoefer, 2020); (ix) the aggregate share of post-match hiring cost to total hiring costs is 70 percent (Yashiv, 2000; Furlanetto and Groshenny, 2016); (x) the average income replacement ratio of 71 percent (Hall and Milgrom, 2008); (xi) the average annual wage growth on job change of 9.0 percent (Hyatt and McEntarfer, 2012b).[14]

Although there is no one-to-one relationship between the parameters and the targeted moments, I briefly describe which targets are informative to pin down their values. First, ψ is determined by targets (i), (ii), and (iii). Specifically, target (i) induces $\tilde{f}(\theta) = (1 -$

[13] See their table 3. They report, in the SIPP sample over 2008-2016, the monthly probability of negative base wage change for job stayers is 0.6 percent.

[14] Hyatt and McEntarfer (2012b), based on job-to-job flow statistics derived from the LEHD, document that median earnings change from direct job switches is around 7 percent to 11 percent.

$\sigma)(1-\delta)f(\theta) = 0.45$ and target (ii) induces $\tilde{q}(\theta) = (1-\sigma)(1-\delta)q(\theta) = 0.71$, which yield the steady-state labor market tightness $\theta = 0.45/0.71 = 0.634$. From the steady-state condition for (3), target (iii) requires $u = (\sigma + (1-\sigma)\delta)/(\sigma + (1-\sigma)\delta + \tilde{f}(\theta)) = 0.07$. Given target (i), it gives the steady-state value of the worker's transition probability from employment to unemployment $\sigma + (1-\sigma)\delta$ of 3.387 percent. Thereby, $f(\theta) = 0.466$, and $q(\theta) = 0.735$, leading to $\psi = 1.385$ from the matching function.

Then, targets (iv), (v), and (vii) are used to pin down σ, δ, and p_{HL}. In the model, the steady-state quit rate is $e_L(w_{oL})f(\theta)\ell_L/\ell$. Thus, target (iv) requires

$$\frac{e_L(w_{oL})f(\theta)\ell_L}{\ell} = \frac{e_L(w_{oL})f(\theta)}{1 + A(p_{HL})e_L(w_{oL})f(\theta)} = 0.025, \tag{15}$$

where $A(p_{HL}) = (1-\sigma)(1-\delta)(1-p_{HL})/[1-(1-\sigma)(1-\delta)(1-p_{HL})]$. In the steady-state, mass $(1 - e_L(w_{oL})f(\theta))\ell_L + \ell_H$ of workers are job stayers, and mass $p_{HL}\ell_H$ of job stayers experience a wage cut. Thus, target (v) requires

$$\frac{p_{HL}\ell_H}{(1 - e_L(w_{oL})f(\theta))\ell_L + \ell_H} = \frac{p_{HL}A(p_{HL})e_L(w_{oL})f(\theta)}{1 - (1 - A(p_{HL}))e_L(w_{oL})f(\theta)} = 0.006. \tag{16}$$

Solving (15) and (16) yields $p_{HL} = 0.011$ and $e_L(w_{oL}) = 0.116$, implying the steady-state value of the vacancy stock $v = \theta(u + e_L(w_{oL})\ell_L) = 0.076$. Then, σ and δ are separately determined from target (vii). The steady-state condition for (4) implies

$$\frac{v_{in} - v_n}{v_{in}} = \frac{(1-\delta)\left[\sigma(1-u) + (1-\sigma)e_L(w_{oL})f(\theta)\ell_L\right]}{(\delta + \tilde{q}(\theta))v} = 0.56.$$

This gives the match-separation rate $\sigma = 0.010$ and the job-destruction rate $\delta = 0.024$.

The remaining parameters are ς, ξ, a_k, ζ_v, φ_H, b, and ϵ_H. Targets (vi), (vii), and (ix) require $\mathcal{S}(e_L(w_{oL}))/Y = 0.04$, $-\partial v_n/\partial v = 0.1183$, and $q(\theta)\varphi_H/(\zeta_v + q(\theta)\varphi_H) = 0.7$, respectively. Target (x) requires $b/\bar{w} = 0.71$, which is used to pin down the value for b. Target (xi) requires $\bar{w}_e/w_{oL} = 1.09$ with $\bar{w}_e = (w_e + 11 \times w_{oH})/12$. As a result, I obtain $\varsigma = 0.062$, $\xi = 1.700$, $a_k = 0.101$, $\zeta_v = 0.168$, $\varphi_H = 0.535$, $b = 0.703$, and $\epsilon_H = 1.075$.

4.3 Steady State

I highlight the several features of the steady state of the model. First, in the calibrated model, the conditions for the convexity of the bargaining set (Proposition 1) are satisfied ($\mathcal{S}'''(e_L) = \varsigma\xi(1-\xi) = -0.073 < 0$, $w_{oL} = 0.948 \in (0,1)$, and $J'_{oL}(w_{oL}) = -0.901$). Second, upgrading job match quality leads to an wage increase: $w_{oL} = 0.948 < 1 < w_{oH} = 1.011 < w_e = 1.276$. Third, the efficiency wage consideration against worker turnover increases the worker's effective bargaining power but its magnitude is not substantial in my calibration: $r(w_{oL,t}) = 0.512 > \eta = 0.5$. Fourth, the share of low-paying employment to total employment is $\ell_L/\ell = 0.429/0.930 = 0.461$, implying that 46 percent of employed workers are conducting on-the-job search in the steady state. Fifth, the share of job changers to total new hires is 42 percent in the steady state, in line with the CPS evidence (Fallick and Fleischman, 2004). Finally, the firm's surplus from a match is higher when matched to an employed worker than an unemployed worker: $M_e - \hat{V} = 1.137 > M_u - \hat{V} = 0.505$, indicating an increase in the share of employed job seekers raises the value of a vacancy (see (7)).

4.4 Cyclical Behavior

Table 3 compares the cyclical property of major labor market variables between the US data (Panel A) and the date generated from the model with different specifications (Panel B). Below, the monthly data is averaged at the quarterly level because output is only available quarterly. To extract the cyclical fluctuations, I take the difference between the log of each variable and its Hodrick–Prescott (HP) filtered trend with smoothing parameter of $1,600$.

4.4.1 US Data

Panel A in Table 3 reports the empirical (i) standard deviations relative to standard deviation of real output, labeled by relative standard deviations (ii) autocorrelations, and (iii) correlations with real output of the five variables: real output (Y, the second column); average real wage per worker ($\bar{w}$, the third column); unemployment (u, the fourth column); vacancy rate (v, the fifth column), and job-finding probability for unemployed workers (f, the sixth column) over a sample period of 224 quarters from 1964:I and 2019:IV. The measurement of each variable is as follows. The real output is measured by real output in the nonfarm

Table 3: Business Cycle Statistics for Major Labor Market Variables

	Y	$\bar{w}$	u	v	f
Panel A			**US Data**		
			Sample period: 1964:I-2019:IV		
Relative Standard Deviation	1.000	0.533	5.705	6.666	4.167
Autocorrelation	0.873	0.891	0.925	0.923	0.842
Correlation with Output	1.000	0.349	−0.858	0.907	0.811
Panel B			**Baseline Model**		
			[1] Benchmark: Endogenous OJS and Full RH		
Relative Standard Deviation	1.000	0.580 (0.56, 0.60)	4.361 (3.07, 6.22)	8.062 (6.03, 11.20)	4.996 (3.93, 7.06)
Autocorrelation	0.872 (0.83, 0.91)	0.915 (0.88, 0.94)	0.923 (0.90, 0.94)	0.890 (0.86, 0.92)	0.915 (0.89, 0.94)
Correlation with Output	1.000	0.964 (0.96, 0.97)	-0.819 (-0.88, -0.74)	0.912 (0.85, 0.94)	0.922 (0.87, 0.95)
			[2] Endogenous OJS and No RH		
Relative Standard Deviation	1.000	0.826 (0.82, 0.83)	1.901 (1.62, 2.21)	3.122 (2.92, 3.35)	2.160 (2.05, 2.31)
Autocorrelation	0.820 (0.76, 0.86)	0.815 (0.76, 0.86)	0.879 (0.84, 0.91)	0.773 (0.71, 0.82)	0.844 (0.80, 0.88)
Correlation with Output	1.000	1.000 (1.00, 1.00)	-0.879 (-0.91, -0.84)	0.957 (0.95, 0.96)	0.992 (0.99, 0.99)
			[3] Constant OJS and Full RH		
Relative Standard Deviation	1.000	0.757 (0.74, 0.77)	2.556 (2.09, 3.10)	2.894 (2.69, 3.17)	2.814 (2.55, 3.22)
Autocorrelation	0.822 (0.77, 0.87)	0.848 (0.80, 0.89)	0.910 (0.88, 0.93)	0.868 (0.83, 0.90)	0.900 (0.87, 0.92)
Correlation with Output	1.000	0.995 (0.99, 1.00)	-0.796 (-0.84, -0.74)	0.975 (0.97, 0.98)	0.931 (0.91, 0.95)
			[4] Constant OJS and No RH		
Relative Standard Deviation	1.000	0.836 (0.83, 0.84)	2.038 (1.73, 2.38)	2.214 (2.12, 2.32)	2.291 (2.15, 2.50)
Autocorrelation	0.813 (0.76, 0.86)	0.813 (0.76, 0.86)	0.882 (0.85, 0.91)	0.781 (0.72, 0.83)	0.853 (0.81, 0.89)
Correlation with Output	1.000	1.000 (1.00, 1.00)	-0.861 (-0.89, -0.82)	0.980 (0.98, 0.98)	0.985 (0.98, 0.99)

Note: Relative standard deviation represents the standard deviation of each variable relative to standard deviation of (detrended) real output. "RH" and "OJS" represent replacement hiring and on-the-job search, respectively. Y is real output, $\bar{w}$ is average wage per worker, u is unemployment, v is vacancy, and f is the job-finding probability for unemployed workers. Bootstrapped 95% confidence intervals are reported in parentheses. All variables are reported in quarterly averages of monthly observations.

business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts. The average real wage per worker is measured by the average per worker earnings of production and non-supervisory employees in the private sector, deflated with Personal Consumption Expenditures Price Index, as in GHT.[15] Unemployment is

[15]The average per worker earnings of production and non-supervisory employees in the private sector is available from 1964:I.

measured by the number of civilian unemployment 16 years and older, constructed by BLS from the CPS. Vacancy is measured by the composite Help-Wanted Index that combines print and online help-wanted advertising following the procedure proposed by Barnichon (2010) and Job Openings in JOLTS. The job-finding probability for unemployed workers is the quarterly average of the monthly job-finding probabilities measured by Shimer's (2005a) approach: $f_t = 1 - (u_{t+1} - u^s_{t+1})/u_t$, where u_t is the number of unemployed workers at the beginning of a month t and u^s_{t+1} is the number of workers unemployed for 0 to 4 weeks at the beginning of a month $t+1$. Namely, u^s_{t+1} counts the number of workers who are newly unemployed between the beginning of a month t and the beginning of the next month $t+1$.

In the US data, the standard deviations of unemployment, vacancies, and the job-finding probability are about 5.71, 6.67, and 4.17 times the standard deviation of real output, respectively, while the standard deviations of the average real wage per worker is about 53 percent of the standard deviation of real output.

4.4.2 Model Simulations

I generate 500 samples of the quarterly data from the model for 224 quarters. To examine the roles of replacement hiring and procyclical on-the-job search in cyclicality of labor market dynamics, I consider the following four different model specifications:

Specification 1 (Benchmark)**.** Employed workers are able to flexibly choose their job search intensity and firms are fully able to re-post a vacancy for replacement hiring ($\gamma = 1$).

Specification 2 (Endogenous On-the-Job Search and No Replacement Hiring)**.** Employed workers are able to flexibly choose their job search intensity, but firms are not able to re-post a vacancy for replacement hiring ($\gamma = 0$).

Specification 3 (Exogenous On-the-Job Search and Full Replacement Hiring)**.** Employed workers conduct on-the-job search with constant intensity: $e_{L,t} = e_L$ and firms are fully able to re-post a vacancy for replacement hiring ($\gamma = 1$).

Specification 4 (Exogenous On-the-Job Search and No Replacement Hiring)**.** Employed workers conduct on-the-job search with constant intensity: $e_{L,t} = e_L$ and firms are not able to re-post a vacancy for replacement hiring ($\gamma = 0$).

That is, relative to Specification 4, Specification 3 considers only replacement hiring, Specification 2 considers only endogenous employed search, and Specification 1 considers both features.

The Panel B of Table 3 reports the business cycle statistics of the five variables in the date generated from the model, comparing the empirical counterparts. It shows that, compared to the other three specifications, the benchmark specification produces an enhanced and persistent response of output, unemployment, vacancies and job-finding probability for unemployed workers, and, given the statistical uncertainty arising from the finite sample period, the calibrated model is able to reproduce the business cycle property comparable to the US data. I emphasize that considering both features (endogenous on-the-job search and replacement hiring) are essential to the enhanced unemployment volatility and that considering only one of them is inadequate; for example, on average over the 500 sample, the ratio of the standard deviation of unemployment to that of output is 4.361 in the benchmark, about 2.3 times larger than the ratio 2.038 in Specification 4. In contrast, the ratios in Specification 2 and 3 is 1.901 and 2.556, about 0.93 and 1.23 times the ratio in Specification 4.[16] Similarly, incorporating both features more than double cyclical volatility of vacancies and job-finding probability for unemployed workers.

Turning to the average wage per worker, I find that the presence of endogenous on-the-job search and replacement hiring reduces the relative standard deviation by 30 percent—it is 0.580 in the benchmark, while it is 0.836 in Specification 4. It is important to note that considering only endogenous on-the-job search (Specification 2) and considering only replacement hiring (Specification 3) have a limited effect on reducing the relative standard deviation of the average wage, indicating that combining both features is crucial to enhanced cyclical volatility of unemployment and reduced cyclical volatility of the average wage.

Table 4 provides a more detailed results regarding the cyclical volatility of wages in the calibrated model. The second column reports the relative standard deviation of the average wage per worker (as reported in the third column of Table 3). Then, the third and fourth columns report the relative standard deviations of the average wage per incumbent workers ($\bar{w}_t^{inc}$) and the average wage per new hires ($\bar{w}_t^{new}$), respectively. They show that

[16]The reduced unemployment volatility in Specification 2 is not unusual in random search models with on-the-job search particularly when the crowding-out effect of employed job search on unemployed job seekers is strong, as emphasized in Pissarides (1994).

Table 4: Relative Standard Deviation

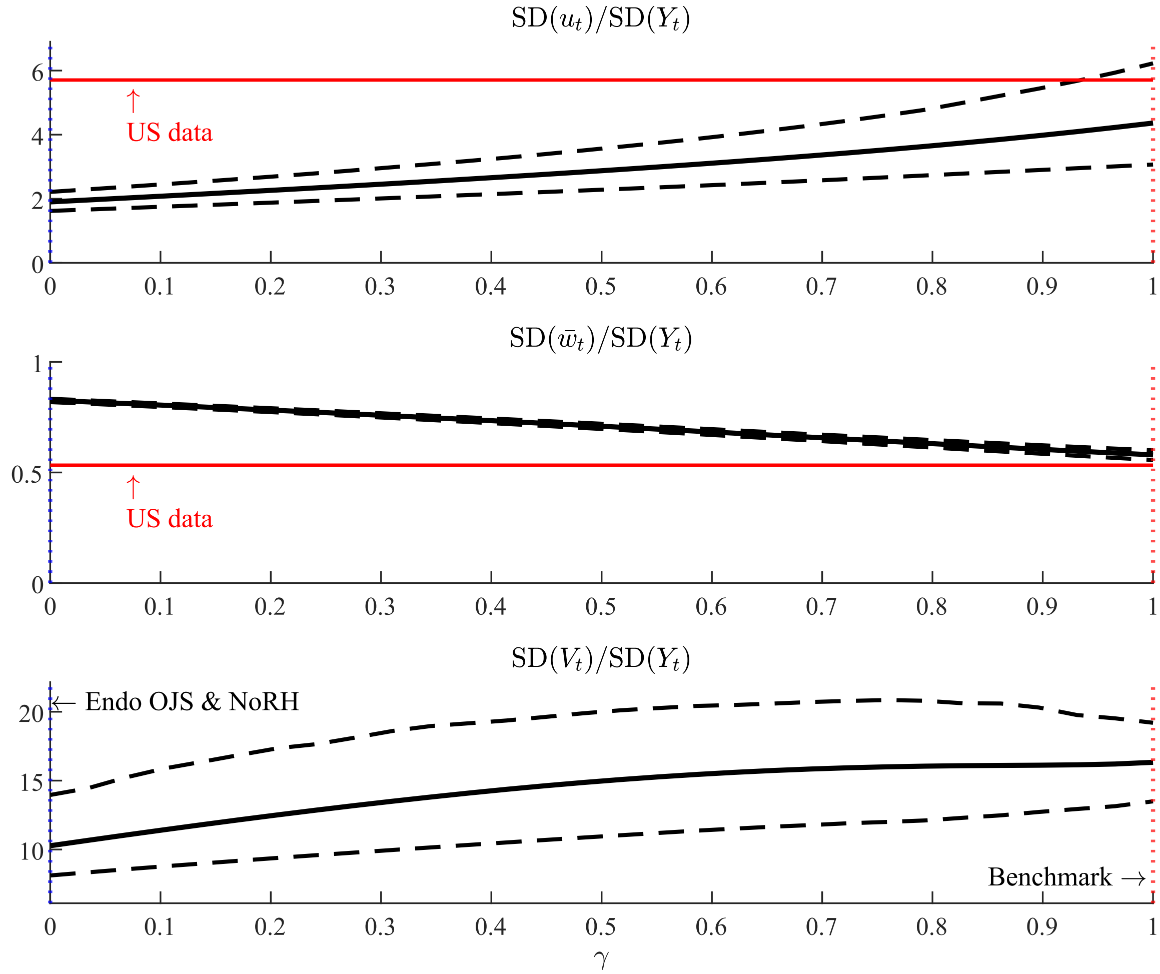
	$\bar{w}$	$\bar{w}^{inc}$	$\bar{w}^{new}$	w_{oH}	w_{oL}	w_e	V
[1] Benchmark	0.580 (0.56, 0.60)	0.549 (0.52, 0.57)	0.782 (0.75, 0.81)	0.553 (0.53, 0.57)	0.273 (0.26, 0.29)	0.209 (0.18, 0.24)	16.321 (13.49, 19.19)
[2] Endo OJS & No RH	0.826 (0.82, 0.83)	0.792 (0.78, 0.80)	1.089 (1.07, 1.11)	0.779 (0.77, 0.79)	0.733 (0.72, 0.74)	0.780 (0.77, 0.79)	10.272 (8.11, 13.96)
[3] Exo OJS & Full RH	0.757 (0.74, 0.77)	0.747 (0.73, 0.76)	0.812 (0.80, 0.82)	0.781 (0.77, 0.79)	0.654 (0.64, 0.66)	0.701 (0.68, 0.72)	7.015 (6.27, 7.85)
[4] Exo OJS & No RH	0.836 (0.83, 0.84)	0.821 (0.81, 0.83)	0.922 (0.91, 0.93)	0.828 (0.82, 0.83)	0.775 (0.77, 0.78)	0.819 (0.81, 0.83)	5.154 (4.69, 5.69)

Note: Relative standard deviation represents the standard deviation of each variable relative to the standard deviation of output. "RH" and "OJS" denote replacement hiring and on-the-job search, respectively. $\bar{w}$ is average wage per worker, $\bar{w}^{inc}$ is the average wage of incumbent workers, and $\bar{w}^{new}$ is the average wage of new hires. Bootstrapped 95% confidence intervals are reported in parentheses. All variables are reported in quarterly average of the monthly observations.

(i) the presence of endogenous on-the-job search and replacement hiring reduces the cyclical volatility of the average wages of both incumbent workers and new hires, and (ii) the average wage of new hires is more volatile than that of incumbent workers, in line with a volume of household survey data evidence (e.g., Bils, 1985). These average wages are affected by the changes in composition of workers. Instead, the fifth to seventh columns of Table 3 report the relative standard deviations of the wages paid to each job ($w_{oH,t}$, $w_{oL,t}$, and $w_{e,t}$), which are more relevant to firms' labor costs. It is shown that, in the presence of endogenous on-the-job search and replacement hiring, wages paid to all jobs are much less volatile than otherwise and the sluggish wage dynamics are particularly pronounced for workers in low-paying jobs and new hires. The last column reports the relative standard deviation of the value of a vacant job V_t, showing that it is much more volatile in the benchmark case than the other three. In sum, it is shown that the combination of endogenous on-the-job search and replacement hiring (i) enhances the cyclical volatility of labor market flows and employment and unemployment stocks (ii) reduces the cyclical volatility of wages particularly for workers in low-paying jobs and new hires, and (iii) increases the cyclical volatility of the value of a vacant job.

Figure 1 visualizes how the possibility of replacement hiring affects the cyclical volatility of unemployment, the average wage, and the value of a vacant job. It clearly shows that, in the presence of endogenous on-the-job search, the greater possibility of replacement hiring leads to the larger cyclical volatility of unemployment and smaller cyclical volatility of the average wage. Hence, with respective to γ, my model generates an inverse relationship

Figure 1: Possibility of Replacement Hiring and Cyclical Volatility



Note: The horizontal axis represents the value of γ (the possibility of replacement hiring). Top, middle, and bottom panels display the standard deviations of unemployment, the average wage per worker, the value of a vacancy, relative to the standard deviation of output, respectively. The cases with $\gamma = 0$ and $\gamma = 1$ correspond to Specification 2 and Specification 1, respectively The red horizontal lines in the top and middle panels are empirical counterparts.

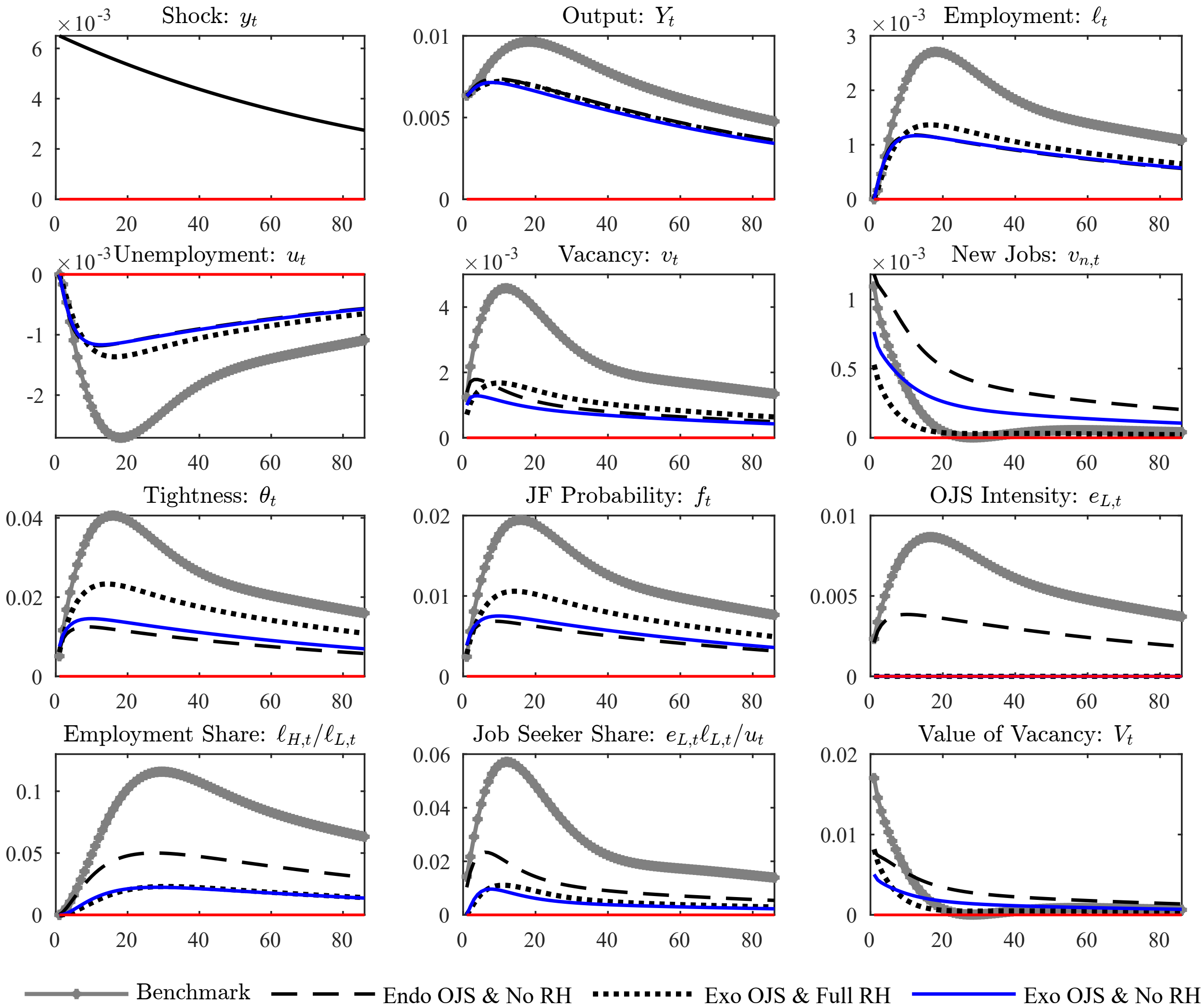
between the cyclical volatility of unemployment and the average wage as in many models in the literature on the unemployment volatility puzzle (e.g., Hall, 2005; Gertler and Trigari, 2009; Christiano, Eichenbaum, and Trabandt, 2016, and others). It is important to note that, as shown in the bottom panel of Figure 1, an increase in γ results in an increase in the cyclical volatility of V_t. As will be detailed in Sections 4.4.3 and 4.4.4, the strong cyclicality of V_t is the key to generate an inverse relationship.

4.4.3 Impulse Responses

To better understand how my model generates both enhanced cyclical volatility of unemployment and sluggish cyclical behavior of wages, I display the impulse-response functions (IRFs) for output and the selected labor market variables to a positive technology shock in Figure 2. It shows that the combination of endogenous on-the-job search and replacement hiring leads to significantly enhanced and persistent responses of output, employment, unemployment, vacancy, labor market tightness, job-finding probability for unemployed workers. Note that, in my calibration, considering procyclical on-the-job search only has almost zero net impact on the unemployment response since it stimulates creation of new jobs but crows out unemployed workers in job searches, as pointed out in the previous studies (Pissarides, 1994; Nagypál, 2006; Martin and Pierrard, 2014). The presence of replacement hiring reduces the crowding-out effects because, even if a vacancy is matched to an employed worker, another vacancy to replace the quitter is posted, and this process continuously occurs until a vacancy is matched to an unemployed worker. While Mercan and Schoefer (2020) and Elsby, Michaels, and Ratner (2020) demonstrate that the presence of replacement hiring generates vacancy-chains effects and amplifies the response of unemployment to productivity shocks, I show that the amplification effect is further enhanced in the presence of procyclical on-the-job search. As can be seen from the IRF for on-the-job search intensity $e_{L,t}$, displayed in the second panel from the bottom on the right, the increased job vacancies for replacement hiring encourage employed workers to search on the job further. This enhances job-to-job transitions, resulting in an enhanced procyclicality of the employment share of better-matched/high-paying employment ($\ell_{H,t}/\ell_{L,t}$) and the share of employed job seeker ($e_{L,t}\ell_{L.t}/u_t$), in line with evidence documented by Haltiwanger et al. (2018) and Moscarini and Postel-Vinay (2008, 2012). As job searches by employed workers become more intensive, vacant firms have more chances to be matched to an experienced worker, so the value of a vacancy V_t increases, as shown in the figure on the lower right. Recall that the value of a vacancy is the value of the firm's outside option in wage negotiations, and thus an increase in its value places downward pressure on the equilibrium wages, as shown in Section 3.

Then, the IRFs for wages are displayed in Figure 3. As can be seen from the top panel, the response of the average wage is much weaker in the benchmark than in the other three. In particular, the combination of endogenous on-the-job search and replacement hiring

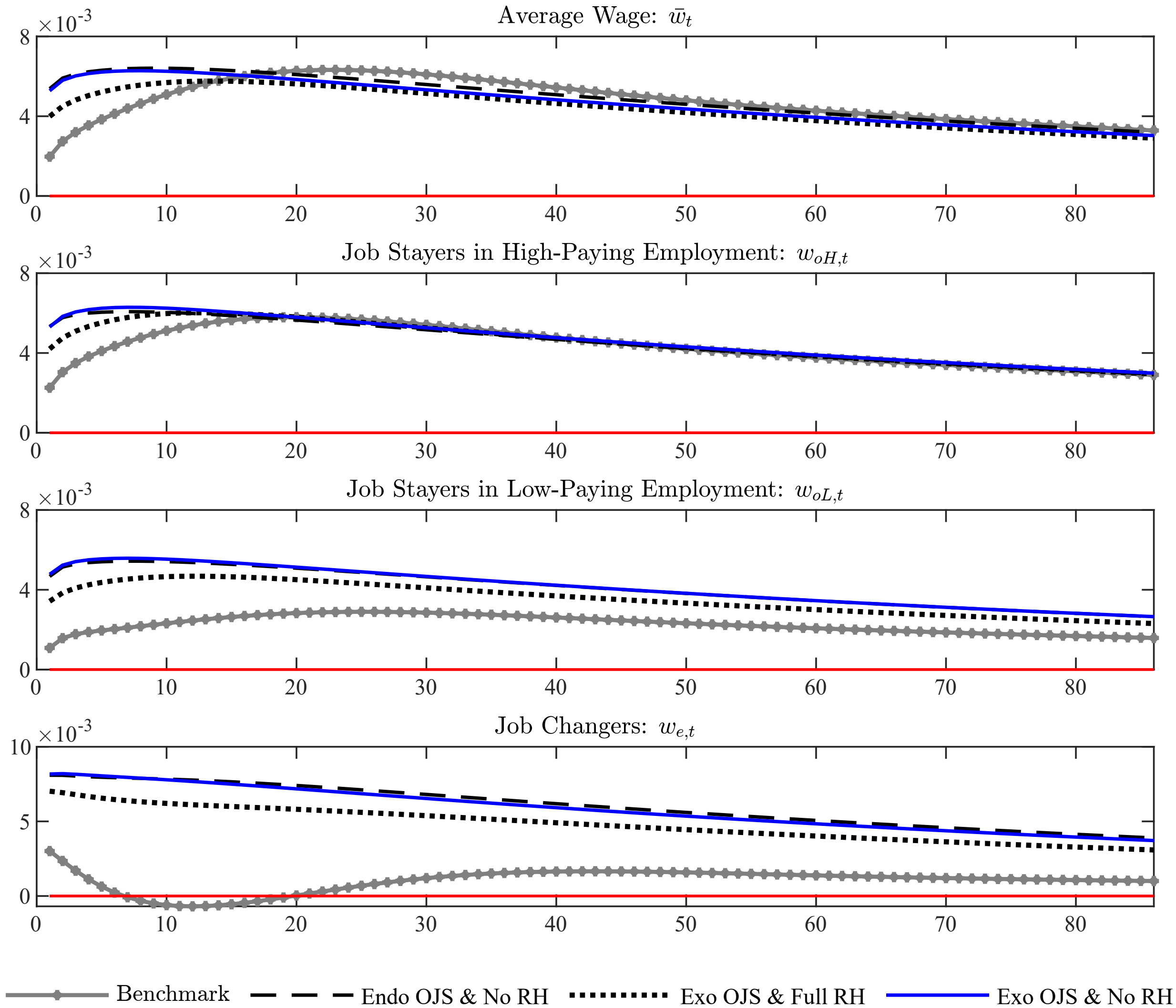
Figure 2: Impulse Responses to Productivity Shock: Output and Labor Market Variables



Note: Each panel plots the impulse-response function to a positive technology shock of one standard deviation with marked solid lines representing the benchmark case, broken lines representing the case in which on-the-job search is endogenous but replacement hiring is impossible, dotted lines representing the case in which on-the-job search is exogenous but replacement hiring is possible, non-marked solid lines representing the case in which on-the-job search is exogenous and replacement hiring is impossible. JF probability denotes job-finding probability. The horizontal axis is the number of months after the realization of the shock and the vertical axis is deviation from the steady-state value.

almost halves the initial impact on it. The bottom three panels show that the calibrated model exhibits sluggish wage responses for all workers, and the sluggish wage responses are particularly pronounced for job stayers in low-wage jobs and job changers. This makes intuitive sense, as the gains from replacing the current worker with a well-matched worker are greater for firms that are poorly matched to workers. In other words, during expansions, firms have a greater chance to replace its poorly-matched worker with a better-matched

Figure 3: Impulse Responses to Productivity Shock: Wages



Note: Each panel plots the impulse-response function to a positive technology shock of one standard deviation with marked solid lines representing the benchmark case, broken lines representing the case in which on-the-job search is endogenous but replacement hiring is impossible, dotted lines representing the case in which on-the-job search is exogenous but replacement hiring is possible, non-marked solid lines representing the case in which on-the-job search is exogenous and replacement hiring is impossible. The impulse responses are displayed in the deviation from the steady-state value. The horizontal axis represents the number of months after the realization of the shock.

worker, which allows them to drive wage negotiations advantageously. The sluggish wages for job stayers spills over into job changers, since job changers are concerned about the difference between their wages and the wages they would have received if they had stayed at their previous jobs.

Table 5: Business Cycle Statistics in the Model with Acyclical Firm's Outside Option Value

	Y	$\bar{w}$	u	v	f
Relative Standard Deviation	1.000	0.710 (0.69, 0.73)	3.399 (2.55, 4.57)	6.052 (4.78, 8.60)	3.770 (3.14, 4.86)
Autocorrelation	0.847 (0.79, 0.89)	0.817 (0.76, 0.87)	0.941 (0.92, 0.96)	0.917 (0.89, 0.94)	0.936 (0.91, 0.95)
Correlation with Output	1.000	0.994 (0.99, 1.00)	-0.690 (-0.77, -0.59)	0.908 (0.85, 0.94)	0.825 (0.76, 0.87)

Note: Relative standard deviation represents the standard deviation of each variable relative to standard deviation of (detrended) real output. Y is real output, $\bar{w}$ is average wage per worker, u is unemployment, v is the vacancy rate, and f is the job-finding probability for unemployed workers. Bootstrapped 95% confidence intervals are reported in parentheses. All variables are reported in quarterly averages of monthly observations.

4.4.4 Roles of Procyclical Firm's Outside Option Value

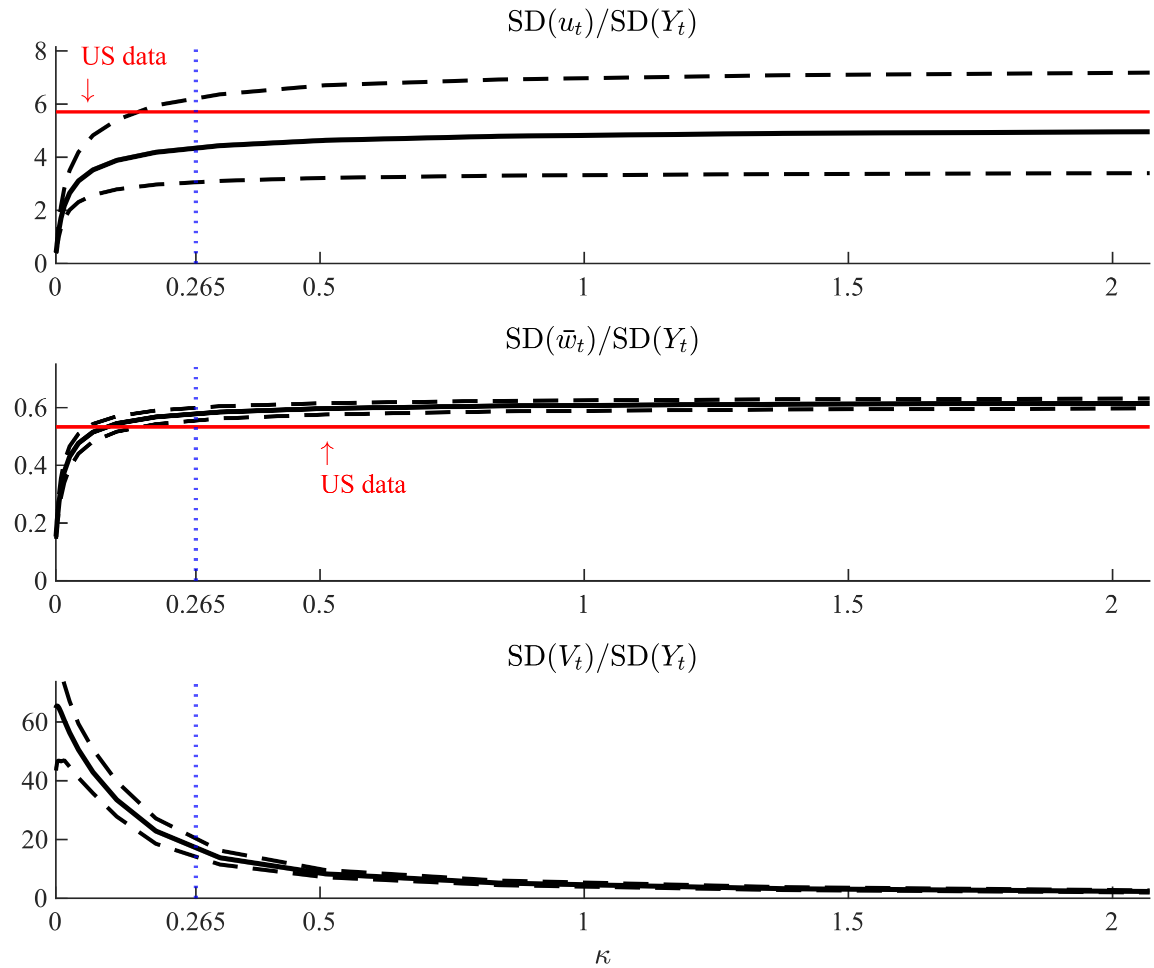
In this section, I further investigate roles of the cyclical behavior of the firm's outside option value in the cyclical behavior of wages. In the model presented in Section 2, firms considered the value of a vacant job to be their outside option value in wage negotiations. Here, I modify this assumption to quantify the extent to which cyclical fluctuations in the value of firms' outside option value contribute to reducing the cyclical volatility of wages. Specifically, I simulate how the model would behave if firms viewed the steady-state value of a vacant job $V = a_k$ as their outside option value in wage negotiations. Namely, it allows for cyclical variations in the value of a vacant job, while firms' outside option value is assumed to be acyclical.

Table 5 reports the business cycle statistics generated from the model. It shows that, if firms' outside option value was acyclical, the average wage per worker would be still volatile even in the presence of endogenous on-the-job search and replacement hiring. Besides, compared with Table 3, it is shown that procyclical fluctuations in firms' outside option value play a role in enhancing unemployment volatility.

4.4.5 Elasticity of Job Creation

The above analysis reveals that the procyclical nature of the value of vacancy plays a key role in increasing unemployment volatility and decreasing wages volatility. While I have focused on how the nature of hiring and job search activity affects the cyclical behavior of the model, in this section, I examine how the nature of entry to job creation affects it.

Figure 4: Elasticity of Job Creation and Cyclical Volatility



Note: The horizontal axis represents the value of κ (the elasticity of entry to job creation). Top, middle, and bottom panels display the standard deviations of unemployment, the average wage per worker, the value of a vacancy, relative to the standard deviation of output, respectively. The red horizontal lines in the top and middle panels are empirical counterparts.

Figure 4 shows how relative standard deviations of unemployment, the average wage per worker, and the value of a vacancy vary with the elasticity of entry to job creation governed by parameter κ. It indicates that, holding other parameters fixed, the higher elasticity of entry to job creation leads to greater cyclical volatility of both unemployment and the average wage. This is because as entry becomes more elastic, the creation of new jobs becomes more procyclical and unemployment fluctuates more, while vacancies for replacement hiring and new jobs become more substitute, resulting in less fluctuation in the value of a vacancy. In the extreme case with $\kappa \to \infty$, aggregate vacancies are determined so that the the value of a vacancy becomes constant $V_t = a_k$.

5 Concluding Remarks

In this paper, I have shown that the Diamond-Mortensen-Pissarides style search-and-matching model featuring inelastic entry to job creation, procyclical on-the-job searches, and replacement hiring is able to generate large cyclical unemployment volatility and sluggish cyclical behavior of wages. The inelastic entry to job creation opens up for cyclical variations in the value of a vacancy, which is the value of firm's outside option in wage negotiations. The presence of on-the-job searches and replacement hiring contributes to increasing procyclicality of the firm's outside option value.

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Appendices

APPENDIX A Proofs

Proof of Proposition 1. From the value functions presented in Section 2, I derive

$$\epsilon_t(w_{oL,t}) \equiv E'_{oL,t}(w_{oL,t}) = 1 - e_{L,t}(w_{oL,t})f(\theta_t) \in (0,1], \tag{A.1}$$

$$\epsilon'_t(w_{oL,t}) = -e'_{L,t}(w_{oL,t})f(\theta_t),$$

$$\mu_t(w_{oL,t}) \equiv -J'_{oL,t}(w_{oL,t}) = 1 - e_{L,t}(w_{oL,t})f(\theta_t) + e'_{L,t}(w_{o,t})f(\theta_t)G_t > 0,$$

and

$$\mu'_t(w_{oL,t}) = -\, 2e'_{L,t}(w_{oL,t})f(\theta_t) + e''_{L,t}(w_{o,t})f(\theta_t)G_t,$$

where $G_t = y_t\epsilon_L - w_{oL,t} + (1-\sigma)\gamma\mathbb{E}_t[\Lambda_{t,t+1}(J_{oL,t+1} - V_{t+1})]$.

Then, to prove Proposition 1, it is useful to prove the following lemma:

Lemma 1. *The bargaining set is convex if $\epsilon_t(w_{oL,t}) \equiv E'_{oL,t}(w_{oL,t}) > 0$, $\mu_t(w_{oL,t}) \equiv -J'_{oL,t}(w_{oL,t}) > 0$, and $\mu_t(w_{oL,t})\epsilon'_t(w_{oL,t}) - \epsilon_t(w_{oL,t})\mu'_t(w_{oL,t}) \leq 0$.*

Proof of Lemma 1. Let the total surplus of this bargaining game denote $S_{oL}(w) = (J_{oL}(w) - V) + (E_{oL}(w) - E_u)$. Below, let $S_F(w) = J_{oL}(w) - V$ be the firm's surplus and $S_W(w) = E_{oL}(w) - E_u$ be the worker's surplus.

In general, they are be rewritten as

$$S_F(w) = z - f(w)$$
$$S_W(w) = x + g(w),$$

where f and g are continuous and twice differentiable real functions. Then, to show Lemma 1, we need to show that if the following conditions:

$$f'(w) > 0, \tag{A.2}$$

$$g'(w) > 0, \tag{A.3}$$

and

$$f'(w)g''(w) - g'(w)f''(w) \leq 0 \tag{A.4}$$

are satisfied, the set $\mathbb{W} = \{(S_F, S_W) \in \mathbb{R}^2_+ \mid S_F(w_F) + S_W(w_W) \leq S_{oL}(w_F), w_W \in [0, w_F]\}$ is convex.

From (A.2) and (A.3), $S_F(w)$ is strictly decreasing in w and $S_W(w)$ is strictly increasing in w. Thus, define

$$S_F(S_W) = z - f(g^{-1}(S_W - x)),$$

and it is enough to show that $S'_F(S_W) \leq 0$ and $S''_F(S_W) \leq 0$.

First, it is clear that

$$S'_F(S_W) = -f'(g^{-1}(\tilde{S}_W))\frac{1}{g'(g^{-1}((\tilde{S}_W))} < 0 \quad \text{(from (A.2) and (A.3))} \quad ,$$

where $\tilde{S}_W = S_W - x$.

Then, the second derivative is

$$\begin{aligned} S''_F(S_W) = & - f''(g^{-1}(\tilde{S}_W))\left(\frac{1}{g'(g^{-1}(\tilde{S}_W))}\right)^2 + f'(g^{-1}(\tilde{S}_W))\left(\frac{1}{g'(g^{-1}(\tilde{S}_W))}\right)^2 \frac{g''(g^{-1}(\tilde{S}_W))}{g'(g^{-1}(\tilde{S}_W))} \\ = & \left(\frac{1}{g'(g^{-1}(\tilde{S}_W)))}\right)^3 \left[f'(g^{-1}(\tilde{S}_W)))g''(g^{-1}(\tilde{S}_W)) - g'(g^{-1}(\tilde{S}_W))f''(g^{-1}(\tilde{S}_W))\right] \leq 0, \end{aligned}$$

where the last inequality comes from (A.4). □

Then, to prove Proposition 1, it is sufficient to show that, if $\mathcal{S}''' \leq 0$ and $w_{oL,t} \in [0, y_t\epsilon_L]$, the three conditions (i) $\epsilon_t(w_{oL,t}) > 0$, (ii) $\mu_t(w_{oL,t}) > 0$, and (iii) $\mu_t(w_{oL,t})\epsilon'_t(w_{oL,t}) - \epsilon_t(w_{oL,t})\mu'_t(w_{oL,t}) \leq 0$ are satisfied. (A.1) shows that (i) holds. (ii) is assumed to be held. Then, I have to show that (iii) holds.

For that purpose, I use the following lemma:

Lemma 2. *$e_{L,t}(w_{oL,t})$ satisfies:*

(i) $e'_{L,t}(w_{oL,t}) < 0$ if $\mathcal{S}'' \geq 0$

(ii) $e''_{L,t}(w_{oL,t}) \geq 0$ if $\mathcal{S}'' \geq 0$ and $\mathcal{S}''' \leq 0$

Proof of Lemma 2. Rewrite (5)

$$\mathcal{S}'(e_{L,t}(w_{oL,t})) = f(\theta_t)\left(H_{e,t}(w_{e,t}) - C_{L,t}(w_{oL,t})\right). \tag{A.5}$$

Taking (A.5) with respect to $w_{oL,t}$ yields

$$\mathcal{S}''(e_{L,t}(w_{oL,t}))e'_{L,t}(w_{oL,t}) = -f(\theta_t)\underbrace{C'_{L,t}(w_{oL,t})}_{=1}. \tag{A.6}$$

Thus, $e'_{L,t}(w_{oL,t}) = -f(\theta_t)/\mathcal{S}''(e_{L,t}(w_{oL,t})) < 0$ if $\mathcal{S}'' > 0$. Taking (A.6) with respect to $w_{oL,t}$ yields

$$\mathcal{S}''(e_{L,t}(w_{oL,t}))e''_{L,t}(w_{oL,t}) = -\mathcal{S}'''(e_{L,t}(w_{oL,t}))(e'_{L,t}(w_{oL,t}))^2.$$

Rearranging the terms yields

$$e''_{L,t}(w_{oL,t}) = -\frac{\mathcal{S}'''(e_{L,t}(w_{oL,t}))e'_{L,t}(w_{oL,t})^2}{\mathcal{S}''(e_{L,t}(w_{oL,t}))} = -\frac{e_{L,t}(w_{oL,t})\mathcal{S}'''(e_{L,t}(w_{oL,t}))}{\mathcal{S}''(e_{L,t}(w_{oL,t}))}\frac{e'_{L,t}(w_{oL,t})^2}{e_{L,t}(w_{oL,t})}.$$

Therefore $e''_{L,t}(w_{oL,t}) \geq 0$ if and only if $\mathcal{S}''' \leq 0$. □

Then, using lemma 2 and the fact $G \geq 0$ if $w_{oL,t} \leq y_t \epsilon_L$, I obtain

$$\begin{aligned}
&\mu_t(w_{oL,t})\epsilon'_t(w_{oL,t}) - \epsilon_t(w_{oL,t})\mu'_t(w_{oL,t}) \\
&\quad = - e'_{L,t}(w_{oL,t})f(\theta_t)\left[1 - e_{L,t}(w_{oL,t})f(\theta_t) + e'_{L,t}(w_{oL,t})f(\theta_t)G_t\right] \\
&\qquad + \left[1 - e_{L,t}(w_{oL,t})f(\theta_t)\right]\left[2e'_{L,t}(w_{oL,t})f(\theta_t) - e''_{L,t}(w_{oL,t})f(\theta_t)G_t\right] \\
&\quad = e'_{L,t}(w_{oL,t})f(\theta_t)\epsilon_t(w_{oL,t}) - \left[e'_{L,t}(w_{oL,t})^2 f(\theta_t) + \epsilon_t(w_{oL,t})e''_{L,t}(w_{oL,t})\right]f(\theta_t)G_t < 0.
\end{aligned}$$

□

Hence, the firm's marginal gain from decreasing the wage is smaller than the worker's marginal gain from increasing the wage: $-J'_{oL,t}(w_{oL,t}) < E'_{oL,t}(w_{oL,t})$.

Online Appendix (Not for Publication)

APPENDIX B Derivations

B.1 Firm's Surplus

- $J_{oL,t}(w_{oL,t})$ is given by

$$J_{oL,t}(w_{oL,t}) = [1 - e_{L,t}(w_{oL,t})f(\theta_t)](y_t\epsilon_L - w_{oL,t}) + (1-\delta)\begin{pmatrix} \mathbb{E}_t[\Lambda_{t,t+1}J_{oL,t+1}] \\ +\gamma[\sigma + (1-\sigma)e_{L,t}(w_{oL,t})f(\theta_t)]\mathbb{E}_t[\Lambda_{t,t+1}(V_{t+1} - J_{oL,t+1})] \end{pmatrix}. \tag{B.1}$$

- $J_{e,t}(w_{e,t})$ is given by

$$J_{oH,t}(w_{oH,t}) = y_t\epsilon_H - w_{oH,t} + (1-\delta)\begin{pmatrix} \mathbb{E}_t[\Lambda_{t,t+1}J_{oH,t+1}] + \gamma\sigma\mathbb{E}_t[\Lambda_{t,t+1}(V_{t+1} - J_{oH,t+1})] \\ +(1-\sigma)p_{HL}\mathbb{E}_t[\Lambda_{t,t+1}(J_{oL,t+1} - J_{oH,t+1})] \end{pmatrix}. \tag{B.2}$$

- $M_{u,t}$ is given by

$$M_{u,t} = (1-\delta)\left(\sigma\mathbb{E}_t[\Lambda_{t,t+1}(V_{t+1} - J_{oL,t+1})] + \mathbb{E}_t[\Lambda_{t,t+1}J_{oL,t+1}]\right), \tag{B.3}$$

- $M_{e,t}(w_{e,t})$ is given by

$$M_{e,t}(w_{e,t}) = y_t\epsilon_H - w_{e,t} + (1-\delta)\begin{pmatrix} \mathbb{E}_t[\Lambda_{t,t+1}J_{oH,t+1}] + \gamma\sigma\mathbb{E}_t[\Lambda_{t,t+1}(V_{t+1} - J_{oH,t+1})] \\ +(1-\sigma)p_{HL}\mathbb{E}_t[\Lambda_{t,t+1}(J_{oL,t+1} - J_{oH,t+1})] \end{pmatrix}. \tag{B.4}$$

- V_t is given by

$$V_t = -\zeta_v + q(\theta_t)\left[(u_t/s_t)(J_{u,t}(w_{u,t}) - \hat{V}_t) + (1 - u_t/s_t)(J_{e,t}(w_{e,t}) - \hat{V}_t) - \varphi_H\right] + (1-\delta)\mathbb{E}_t[\Lambda_{t,t+1}V_{t+1}]. \tag{B.5}$$

- $\hat{V}_t$ is given by

$$\hat{V}_t = (1-\delta)\mathbb{E}_t[\Lambda_{t,t+1}V_{t+1}]. \tag{B.6}$$

- Use (B.1) and (B.5) to obtain the formula for $F_{oL,t}(w_{oL,t}) = J_{oL,t}(w_{oL,t}) - V_t$:

$$\begin{aligned} F_{oL,t}(w_{oL,t}) =& \zeta_v + [1 - e_{L,t}(w_{oL,t}) f(\theta_t)] (y_t \epsilon_L - w_{oL,t}) \\ & - q(\theta_t) [(u_t/s_t) F_{u,t} + (1 - u_t/s_t) F_{e,t}(w_{e,t}) - \varphi_H] \\ & + (1 - \delta) [1 - \gamma\sigma - \gamma(1 - \sigma) e_{L,t}(w_{oL,t}) f(\theta_t)] \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] . \end{aligned}$$

- Use (B.2) and (B.5) to obtain the formula for $F_{oH,t}(w_{oH,t}) = J_{oH,t}(w_{oH,t}) - V_t$:

$$\begin{aligned} F_{oH,t}(w_{oH,t}) =& \zeta_v + y_t \epsilon_H - w_{oH,t} - q(\theta_t) [(u_t/s_t) F_{u,t} + (1 - u_t/s_t) F_{e,t}(w_{e,t}) - \varphi_H] \\ & + (1 - \delta)[1 - \gamma\sigma - (1 - \sigma) p_{HL}] \mathbb{E}_t [\Lambda_{t,t+1} F_{oH,t+1}] \\ & + p_{HL} (1 - \sigma)(1 - \delta) \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] . \end{aligned}$$

- Use (B.3) and (B.6) to obtain the formula for $F_{u,t} = J_{u,t}(w_{u,t}) - \hat{V}_t$:

$$F_{u,t} = (1 - \sigma)(1 - \delta) \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] .$$

- Use (B.4) and (B.6) to obtain the formula for $F_{e,t}(w_{e,t}) = J_{e,t}(w_{e,t}) - \hat{V}_t$:

$$\begin{aligned} F_{e,t}(w_{e,t}) = y_t \epsilon_H - w_{e,t} + (1 - \delta)[1 - \gamma\sigma - (1 - \sigma) p_{HL}] \mathbb{E}_t [\Lambda_{t,t+1} F_{oH,t+1}] \\ + p_{HL} (1 - \sigma)(1 - \delta) \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] . \end{aligned}$$

- Then, $\mu_{oL,t}$ is given by

$$\begin{aligned} \mu_{oL,t}(w_{oL,t}) =& 1 - e(w_{oL,t}) f(\theta_t) \\ & + e'_t(w_{oL,t}) f(\theta_t) \{ y_t \epsilon_L - w_{oL,t} + (1 - \sigma)(1 - \delta) \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] \} . \end{aligned}$$

  Hence,

$$\begin{aligned} \mu'_{oL,t}(w_{oL,t}) =& - e'(w_{oL,t}) f(\theta_t) \\ & + e''_t(w_{oL,t}) f(\theta_t) \{ y_t - w_{oL,t} + (1 - \sigma)(1 - \delta) \mathbb{E}_t [\Lambda_{t,t+1} F_{oL,t+1}] \} . \end{aligned}$$

B.2 Worker's Surplus

- $E_{oH,t}(w_{oH,t})$ is given by

$$\begin{aligned} E_{oH,t}(w_{oH,t}) =& w_{oH,t} + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oH,t+1}\right)\right] \\ &+ p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{oH,t+1}\right)\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oH,t+1}\right]. \end{aligned} \tag{B.7}$$

- $E_{oL,t}(w_{oL,t})$ is given by

$$E_{oL,t}(w_{oL,t}) = -\mathcal{S}(e_t(w_{oL,t})) + e_t(w_{oL,t})f(\theta_t)H_{e,t}(w_{e,t}) + [1 - e_t(w_{oL,t})f(\theta_t)]C_{L,t} \tag{B.8}$$

- $H_{e,t}(w_{e,t})$ is given by

$$\begin{aligned} H_{e,t}(w_{e,t}) =& w_{e,t} + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oH,t+1}\right)\right] \\ &+ p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{oH,t+1}\right)\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oH,t+1}\right]. \end{aligned} \tag{B.9}$$

- $C_{L,t}$ is given by

$$C_{L,t} = w_{oL,t} + [\sigma + (1-\sigma)\delta]\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{u,t+1} - E_{oL,t+1}\right)\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{oL,t+1}\right]. \tag{B.10}$$

- $E_{u,t}$ is given by

$$E_{u,t} = b + \mathbb{E}_t\left[\Lambda_{t,t+1}E_{u,t+1}\right] + (1-\sigma)(1-\delta)f(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(E_{oL,t+1} - E_{u,t+1}\right)\right] \tag{B.11}$$

- Use (B.7) and (B.11) to obtain the formula for $W_{oH,t}(w_{oH,t}) = E_{oH,t}(w_{oH,t}) - E_{u,t}$:

$$\begin{aligned} W_{oH,t}(w_{oH,t}) =& w_{oH,t} - b + (1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left((1-p_{HL})W_{oH,t+1} + p_{HL}W_{oL,t+1}\right)\right] \\ &- (1-\sigma)(1-\delta)f(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]. \end{aligned}$$

- Use (B.8) and (B.11) to obtain the formula for $W_{oL,t}(w_{oL,t}) = E_{oL,t}(w_{oL,t}) - E_{u,t}$:

$$\begin{aligned} W_{oL,t}(w_{oL,t}) =& [1 - e_{L,t}(w_{oL,t})f(\theta_t)]w_{oL,t} - \mathcal{S}(e_{L,t}(w_{oL,t})) - b \\ &+ (1-\sigma)(1-\delta)(1-f(\theta_t))\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right] \\ &+ e_t(w_{oL,t})f(\theta_t)\left[H_{e,t}(w_{e,t}) - E_{u,t}\right]. \end{aligned}$$

$$W_{oL,t}(w_{oL,t}) = -\mathcal{S}(e_t(w_{oL,t})) + e_t(w_{oL,t})f(\theta_t)\left(H_{e,t}(w_{e,t}) - C_{L,t}\right) + \left(C_{L,t} - E_{u,t}\right),$$

where

$$C_{L,t} - E_{u,t} = w_{oL,t} - b + (1-\sigma)(1-\delta)(1-f(\theta_t))\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right].$$

- Use (B.9) and (B.10) to obtain the formula for $W_{e,t}(w_{e,t}) = E_{e,t}(w_{e,t}) - C_{L,t}$:

$$W_{e,t}(w_{e,t}) = w_{e,t} - w_{oL,t} + (1-p_{HL})(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(W_{oH,t+1} - W_{oL,t+1}\right)\right].$$

B.3 Wage Functions

- Wage bargaining results in that

$$\eta F_{oH,t}(w_{oH,t}) = (1-\eta)W_{oH,t}(w_{oH,t}), \tag{B.12}$$

$$r(w_{oL,t})F_{oL,t}(w_{oL,t}) = (1-r(w_{oL,t}))W_{oL,t}(w_{oL,t}), \tag{B.13}$$

and

$$\eta F_{e,t}(w_{e,t}) = (1-\eta)W_{e,t}(w_{e,t}). \tag{B.14}$$

- (B.12) gives

$$\begin{aligned}
(1-\eta)&\left[w_{oH,t} - b + (1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left((1-p_{HL})W_{oH,t+1} + p_{HL}W_{oL,t+1}\right)\right]\right] \\
&- (1-\eta)(1-\sigma)(1-\delta)f(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right] \\
&= \eta\left[y_t\epsilon_H - w_{oH,t} + \zeta_v - q(\theta_t)\left[(u_t/s_t)F_{u,t} + (1-u_t/s_t)F_{e,t}(w_{e,t}) - \varphi_H\right]\right] \\
&+ \eta(1-\delta)[1-\gamma\sigma-(1-\sigma)p_{HL}]\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oH,t+1}\right] \\
&+ \eta p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oL,t+1}\right],
\end{aligned}$$

and

$$
\begin{aligned}
w_{oH,t} =&(1-\eta)\left[b+\tilde{f}(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]\right] \\
&+\eta y_t\epsilon_H-\eta\left[V_t-(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right] \\
&+(1-\sigma)(1-\delta)p_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta F_{oL,t+1}-(1-\eta)W_{oL,t+1}\right)\right] \\
&+\eta(1-\delta)[1-\gamma\sigma-(1-\sigma)p_{HL}]\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oH,t+1}\right] \\
&-(1-\eta)(1-\sigma)(1-\delta)(1-p_{HL})\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oH,t+1}\right].
\end{aligned}
$$

Then, for the special case $\gamma=1$,

$$
\begin{aligned}
w_{oH,t} =&(1-\eta)\left[b+\tilde{f}(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]\right]+\eta y_t\epsilon_H-\eta\left[V_t-(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right] \\
&+(1-\sigma)(1-\delta)p_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta F_{oL,t+1}-(1-\eta)W_{oL,t+1}\right)\right] \\
&+(1-\sigma)(1-\delta)(1-p_{HL})\mathbb{E}_t\left[\Lambda_{t,t+1}\underbrace{\left(\eta F_{oH,t+1}-(1-\eta)W_{oH,t+1}\right)}_{=0 \text{ from (B.12)}}\right].
\end{aligned}
$$

- (B.13) implies

$$
\begin{aligned}
(1-r(w_{oL,t}))&\left[\begin{array}{l} w_{oL,t}-b+(1-\sigma)(1-\delta)(1-f(\theta_t))\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right] \\ -\mathcal{S}(e_t(w_{oL,t}))+e_t(w_{oL,t})f(\theta_t)\left(H_{e,t}(w_{e,t})-C_{L,t}\right) \end{array}\right] \\
=&r(w_{oL,t})\left[\begin{array}{l} \left[1-e_{L,t}(w_{oL,t})f(\theta_t)\right]\left(y_t\epsilon_L-w_{oL,t}\right) \\ +\zeta_v-q(\theta_t)\left[(u_t/s_t)F_{u,t}+(1-u_t/s_t)F_{e,t}(w_{e,t})-\varphi_H\right] \end{array}\right] \\
&+r(w_{oL,t})(1-\delta)\left[1-\gamma\sigma-\gamma(1-\sigma)e_{L,t}(w_{oL,t})f(\theta_t)\right]\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oL,t+1}\right],
\end{aligned}
$$

and thus

$$
\begin{aligned}
&[1-e_{L,t}(w_{oL,t})f(\theta_t)]w_{oL,t} \\
&=(1-r(w_{oL,t}))\left[b+\mathcal{S}(e_{L,t}(w_{oL,t}))-(1-\sigma)(1-\delta)(1-f(\theta_t))\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]\right] \\
&\quad-(1-r(w_{oL,t}))e_t(w_{oL,t})f(\theta_t)\left(H_{e,t}(w_{e,t})-C_{L,t}\right) \\
&\quad+r(w_{oL,t})\left[1-e_{L,t}(w_{oL,t})f(\theta_t)\right]y_t\epsilon_L-r(w_{oL,t})\left[V_t-(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right] \\
&\quad+r(w_{oL,t})(1-\delta)\left[1-\gamma\sigma-\gamma(1-\sigma)e_{L,t}(w_{oL,t})f(\theta_t)\right]\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oL,t+1}\right].
\end{aligned}
$$

Then, for the special case $\gamma = 1$,

$$\begin{aligned}
&[1-e_{L,t}(w_{oL,t})f(\theta_t)]w_{oL,t} \\
&\quad =(1-r(w_{oL,t}))\left[b+\mathcal{S}(e_{L,t}(w_{oL,t}))+(1-\sigma)(1-\delta)f(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]\right] \\
&\qquad -(1-r(w_{oL,t}))e_t(w_{oL,t})f(\theta_t)\left(H_{e,t}(w_{e,t})-C_{L,t}\right) \\
&\qquad +r(w_{oL,t})\left[[1-e_{L,t}(w_{oL,t})f(\theta_t)]\,y_t\epsilon_L-(1-\delta)(1-\sigma)e_{L,t}(w_{oL,t})f(\theta_t)\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oL,t+1}\right]\right] \\
&\qquad -r(w_{oL,t})\left[V_t-(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}\right]\right].
\end{aligned}$$

- (B.14) implies

$$\begin{aligned}
&(1-\eta)\left[w_{e,t}-w_{oL,t}+(1-p_{HL})(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(W_{oH,t+1}-W_{oL,t+1}\right)\right]\right] \\
&\quad =\eta\left[y_t\epsilon_H-w_{e,t}+(1-\delta)[1-\gamma\sigma-(1-\sigma)p_{HL}]\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oH,t+1}\right]\right] \\
&\qquad +\eta p_{HL}(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oL,t+1}\right],
\end{aligned}$$

and thus

$$\begin{aligned}
w_{e,t} &= \eta y_t\epsilon_H+(1-\eta)w_{oL,t} \\
&\quad +(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta p_{HL}F_{oL,t+1}+(1-\eta)(1-p_{HL})W_{oL,t+1}\right)\right] \\
&\quad +(1-\delta)[1-\gamma\sigma-(1-\sigma)p_{HL}]\eta\mathbb{E}_t\left[\Lambda_{t,t+1}F_{oH,t+1}\right] \\
&\quad -(1-\sigma)(1-\delta)(1-p_{HL})(1-\eta)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oH,t+1}\right].
\end{aligned}$$

Then, for the special case $\gamma = 1$,

$$\begin{aligned}
w_{e,t} &= (1-\eta)\left[w_{oL,t}+(1-\sigma)(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}W_{oL,t+1}\right]\right]+\eta y_t\epsilon_H \\
&\quad +(1-\sigma)(1-\delta)p_{HL}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\eta F_{oL,t+1}-(1-\eta)W_{oL,t+1}\right)\right] \\
&\quad +(1-\sigma)(1-\delta)(1-p_{HL})\mathbb{E}_t\left[\Lambda_{t,t+1}\underbrace{\left(\eta F_{oH,t+1}-(1-\eta)W_{oH,t+1}\right)}_{=0\text{ from (B.12)}}\right].
\end{aligned}$$