The Effects of Trend Inflation on Firm Uncertainty and Price Setting Behavior*

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Abstract

In this paper, I study the effect of changes in the trend inflation rate on the relationship between two firm's decisions: expectation formation and price setting. I develop a new framework in which firms face both menu cost and information friction, and find that there is a strong interaction between the firm's expectation formation and price setting. This interaction endogenously generates heterogeneity in the frequency and size of the price setting, depending on firm uncertainty. I also show that the level of the trend inflation rate is an important factor for the firm's behavior and distribution. Finally, I compute the macroeconomic implication of this interaction. I show numerically that the information frictions amplify the real effects of nominal shocks, while positive inflation rate weakens this amplification mechanism.

Keywords: Menu cost, Rational inattention, Trend inflation, Inflation expectations, Monetary non-neutrality

JEL Classification: D21, D81, D83, E31, E32, E37, E52

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1 Introduction

Firms' expectation formation and pricing behavior are important factors in understanding the effects of monetary policy. Especially, after the Great Recession, the policy rate hit its Zero Lower Bound and policy makers have tried to implement policies aimed at raising the inflation expectations to lower the real interest rates and stimulate economy. However, recent growing literature using firm's survey data finds that the effect of these policies on the firm's belief is limited and uneven across firms: many firms are uninformed about their economic environment and there is a large heterogeneity in firm uncertainty. Furthermore, the other strand of empirical literature shows that there is also a heterogeneity in the firm's price setting and its distribution is influenced by the level of trend inflation rate.

Given these empirical findings, many research questions arise naturally: how are firms' expectation formation and price setting decisions related? What is the macroeconomic implication of the link between these decisions? How changes in the level of trend inflation affect the firm's behavior?

This paper answers these questions by constructing a new theoretical framework in which firms face both menu costs to change their price and costly information about their idiosyncratic productivity. Using this framework, I derive the firm's optimal decisions on information acquisition and price setting, and examine the aggregate implication with different values of the equilibrium inflation rate.

In my model, firms cannot observe their idiosyncratic productivity directly, and instead, estimate its value by obtaining information from noisy signals. I call the conditional variance of the estimates as *firm uncertainty*. Based on the estimated productivity, firms solve their optimization problem. I assume that firms decide how much attention to pay the signal subject to the information cost, which depends on the amount of information included in the signal. This assumption makes firms rationally inattentive to the signal in a sense that sometimes they optimally ignore new signals in updating the estimates.

The first key finding is that there is a close interrelationship between the decision to obtain information and the decision to adjust prices. Both decisions depend on two state variables: the estimated price position and its uncertainty. First, the distance between the own price and the estimated optimal price influences the firm's decisions. When the firm thinks that its own price is close to the desired level, it has no incentive to pay the menu cost to change the price, as with the standard menu cost model, but also does not pay much attention to the signal because of the information cost. On the other hand, if the firm's price is far from the estimated optimal level, the firm tends to acquire costly information to update its estimation and prepares for a price adjustment. Second, the level of uncertainty in the firm's estimates also affects the firm's decisions. I numerically show that the more uncertain a firm is, the more willing it is to pay information costs to make an accurate estimate, and the wider the area of inaction in the pricing policy. Since the price adjustment revises the estimated price position and the information acquisition reduces firm uncertainty, these decisions are linked to each other through changes in state variables. Consequently, with ex ante identical firms, information acquisition choice endogenously generates a heterogeneity in the frequency and size of price changes among firms. This theoretical prediction is new in the literature.

Secondary, I find that each firm's information choice amplifies the real effects of monetary shock. Amplification arises from the following two mechanisms. First, the firm's information choice endogenously generates the leptokurtic distribution of the estimated price gap, which weakens the selection effect observed in the standard menu cost model. Second, uncertainty creates a dispersion in the time it takes to change prices across firms. The firm with low uncertainty updates its estimate very slowly and it takes a long time to change the price. Thus, when the fraction of firms with low uncertainty is large, the real effect of monetary shock becomes persistent. These amplification mechanisms are novel findings of my paper.

Finally, I show that the firm's optimal decisions and their aggregate implications are influenced by the level of trend inflation rate. For example, a positive trend inflation rate has an asymmetric impact on the firm's information acquisition. When inflation is positive, firms can expect the optimal price level to rise in the future, and thus the intensity of attention to idiosyncratic shocks will be lower when raising prices and higher when lowering prices. As a result of the change in the optimal decisions, the level of trend inflation rate also affects the shape of the stationary distribution of the estimated price gap and firm uncertainty. I numerically show that the kurtosis of the price gap distribution depends on the level of inflation rate. With a positive inflation rate, the distribution of the estimated price gap has a fat tail, which strengthens the selection effect. Consequently, the real effect of monetary shock becomes small and short-lived as the inflation rate rises.

This paper also contributes to the literature methodologically. In this paper, I develop a new framework which combines the menu cost and the rational inattentive assumption. To solve not only the stationary equilibrium but also the transition dynamics of this complicated model, I rewrite the firm's problem as a combined stochastic and impulse control problem. Then, the value function of this problem is given by the solution to the so-called Hamilton-Jacobi-Bellman quasi-variational inequality (HJBQVI). I apply a novel numerical scheme to solve this HJBQVI. To the best of my knowledge, this paper is the first research in the theoretical literature that uses the HJBQVI and numerically derives the transition dynamics of the complicated price setting model without any simplified assumptions.

2 Literature Review

There is a large literature that studies the characteristics of the firm's expectations and its implications for the firm's decision making and macroeconomic outcomes using survey data. The common findings in the literature are that firms are on average unaware of their environments and there is a large disagreement in their expectations. Such discrepancies in expectations can be explained not only by the observable characteristics of firms, such as firm size, but also by the difference in how informed on the variables. Moreover, this difference among firms is caused by their willingness to acquire information. These findings support the rational inattention model.

For example, Kumar et al. (2015) and Coibion et al. (2018) use New Zealand's survey data and find that there is a large heterogeneity in the firm's expectations and when given random information, firms with higher ex ante uncertainty are more willing to revise their forecasts than firms with lower ex ante uncertainty. These findings are consistent with the firm's learning technology of my model. In Japan, Kaihatsu et al. (2016) find a heterogeneous reaction of the firm's inflation expectations to monetary policy shocks, which also support the rational inattentive assumption. The relationship between the firm's expectations and its decision making is also found in the empirical papers. Coibion et al. (2020) empirically show the causal effect of information expectations on firms' economic decisions using the survey data of Bank of Italy. Bachmann et al. (2019) show that firm uncertainty affects the frequency and size of the firm's price setting using the Germany survey data.

There is also another strand of literature that studies the firm's price setting behavior using micro firm-level data since the seminal paper of Bils and Klenow (2004). Many papers in the literature support the menu cost model to explain the firm's price setting behavior, especially in the low inflation rate environment (e.g. Nakamura and Steinsson 2008, Gagnon 2009, Alvarez et al. 2019, Higo et al. 2007, Sudo et al. 2014, Watanabe and Watanabe 2018). In those empirical papers, Watanabe and Watanabe (2018) study the relationship between the level of inflation rate and the firm's price setting in Japan. They find that a decline in the inflation rate increases the share of items whose price remains unchanged, which implies the price rigidity increases endogenously due to the level of trend inflation rate.

Compared to the empirical findings, there is a relatively small body of literature that theoretically explains the connection between the firm's expectation formation and price setting behavior[1]. Here, I briefly review the related papers which combine the menu cost

[1]The standard menu cost model without any information friction is studied in, for instance, Golosov and Lucas (2007), Midrigan (2011) or Alvarez and Lippi (2014). For the basic parts of the price setting problem, my paper follows these references.

and information friction to study its implications[2].

Alvarez et al. (2011) and Alvarez et al. (2018) construct a model in which the firm has to pay a fixed cost for both observing the true optimal price and adjusting the price. They assume that firms can perfectly observe their state once they pay the observation cost. As a result, it is optimal for firms to adjust prices at most once and immediately after observing the true value and stay until the next observation. Alvarez et al. (2018) extend this framework and calculate the impulse response of the aggregate output to monetary shock. They find that these two frictions generate large real effects.

This paper differs from their research in the following points. In Alvarez et al. (2011), firms only choose the date of observation and adjust prices only when they perfectly observe the state. In contrast, I assume that firms frequently pay attention to the signals and the intensity of the attention is also chosen by firms and depending on their state. Furthermore, firms always change prices with uncertain estimates. As a result, my framework endogenously generates the interaction between two decisions.

Baley and Blanco (2019) is one of the papers that is most relevant to my research. They construct the menu cost model with imperfect information. By combining information friction and fat-tailed shock in the state process, their model generate some similar results such as cross-sectional variation in price setting decisions to my model. Notice, however, that they assume that firm uncertainty is given by an exogenous process. This paper contributes to their study by endogenizing the firm's information choice.

This paper is also closely related to Yang (2020), which develop a theoretical model of multiproduct firms who face both menu cost and rationally inattentive restrictions. He finds the similar theoretical prediction in firms' optimal decisions. However, he only considers the zero inflation case. In contrast, I derive the effect of trend inflation on the firm's behavior and its macroeconomic implications[3].

The rest of this paper is organized as follows. Section 3 outlines the theoretical model. Section 4 provides the properties of the stationary equilibrium and novel theoretical predictions. In Section 5, I exercise comparative statics to understand the interaction of the firm's information choice and price setting. Section 6 analyzes the macroeconomic consequences of the firm's decision and the impact of the trend inflation rate, and Section 7 concludes.

[2] In addition to the literature I introduce here, there are other related papers such as Bonomo et al. (2021), Gorodnichenko (2008), Mackowiak and Wiederholt (2009) which analyze the interaction between menu costs and information friction.

[3] Alexandrov et al. (2020) examines the effect of trend inflation in the menu cost model without information friction. In this paper, I investigate the effect of trend inflation in a more comprehensive framework.

3 Model

I develop a general equilibrium model in which firms face the fixed price adjustment cost and imperfect information about their idiosyncratic productivity. The model is based on Golosov and Lucas (2007) with an additional assumption on firms' belief formation.

Time is continuous. The economy is populated by a representative household, a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$, and the monetary authority who controls the money supply.

3.1 Representative Household

The preference of a representative household is defined over aggregate consumption goods $c(t)$, labor supply $l(t)$ and real money holdings $M(t)/P(t)$, where $P(t)$ is the aggregate price level:

$$\int_0^\infty e^{-\rho t} \left(\frac{c(t)^{1-\gamma}}{1-\gamma} - \alpha l(t) + \log \frac{M(t)}{P(t)} \right) dt \tag{1}$$

where ρ is the discount rate, γ is the relative risk averse, and α is the disutility of labor. The household chooses the consumption goods, labor supply, and money holdings to maximize its utility subject to the following budget constraints:

$$M(0) + \int_0^\infty Q(t) \left\{ \tilde{\Pi}(t) + W(t)l(t) - r(t)M(t) - P(t)c(t) \right\} dt = 0$$

where $r(t)$ is the nominal interest rate, and thus $r(t)M(t)$ represents the opportunity cost of holding cash, $Q(t) := \exp\left(-\int_0^t r(s)ds\right)$ is the time zero price of a dollar delivered at time t, $W(t)$ is the nominal wage, and $\tilde{\Pi}(t)$ are the firms' profits. Consumption $c(t)$ is composed of a continuum of differentiated goods as

$$c(t) = \left[\int [A_i(t)C_i(t)]^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where ε is the elasticity of substitution, $C_i(t)$ is consumption of a good produced by firm i, and $A_i(t)$ is its idiosyncratic quality[4] which is assumed to follow a Geometric Brownian motion with zero drift term and variance σ^2 with increments that are independent among firms. Let $B_i(t)$ be a standard Brownian motion, logarithmic form of firm i's quality $a_i(t) :=$

[4]The quality of good $A_i(t)$ affects both the marginal cost of firm i and the household's preferences for the good. This assumption about the idiosyncratic shock has been introduced by several papers such as Woodford (2009), Midrigan (2011), Alvarez and Lippi (2014) and Alvarez et al. (2018). It allows us to reduce the dimensionality of the state space.

$\log A_i(t)$ evolves as:

$$da_i(t) = \sigma dB_i(t)$$

I focus on an equilibrium at which the money supply grows at a constant rate μ, then from the household's first-order condition with respect to money holdings , I obtain

$$\frac{e^{-\rho t}}{M(t)} = \lambda Q(t) r(t)$$

where λ is the Lagrange multiplier for the budget constraints. Taking logs and differentiating with respect to time, this equation is written as

$$\dot{r(t)} = r(t)(r(t) - \rho - \mu)$$

This ordinary differential equation has two steady states; $r = 0$ or $r = \rho + \mu > 0$. Since the latter steady state $\rho + \mu$ is unstable, there exists an equilibrium in which the nominal interest rate is constant at the level:

$$r(t) = \rho + \mu \quad \text{for all } t \geq 0 \tag{3}$$

$$Q(t) := \exp\left(-\int_0^t r(s)ds\right) = e^{-(\rho+\mu)t} \tag{4}$$

From the first-order condition with respect to labor supply, the nominal wage $W(t)$ is proportional to the money supply $M(t)$ and thus, its growth rate is μ for all time $t \geq 0$. Moreover, from the first-order conditions with respect to $l(t), c(t)$ and $C_i(t)$, I obtain the demand function for firm i's good $C_i(t)$:

$$C_i(t) = A_i(t)^{\varepsilon-1}\left(\frac{P_i(t)}{P(t)}\right)^{-\varepsilon} c(t) \tag{5}$$

3.2 Monopolistic Firms

There is a continuum of firms producing differentiated goods, indexed by $i \in [0,1]$. Each firm produces goods $C_i(t)$ using labor $l_i(t)$ based on the linear technology: $C_i(t) = l_i(t)/A_i(t)$ and sets price $P_i(t)$. I assume that if the firm i adjusts its nominal price at time t, it has to pay the menu cost $\kappa W(t)$, where the parameter κ is the hours of labor needed to change price. Given $C_i(t)$ and $P_i(t)$, a firm i's instantaneous nominal profit $\hat{\Pi}_i(t)$ is written as

$$\hat{\Pi}_i(t) := P_i(t)C_i(t) - W(t)l_i(t) = C_i(t)[P_i(t) - W(t)A_i(t)]$$

and it maximizes the expected sum of $\hat{\Pi}_i(t)$, discounted at $Q(t)$.

It is convenient to define the firm's optimal price $P_i^*(t)$ that maximizes the instantaneous profit when both price adjustment friction and information friction do not exist:

$$P_i^*(t) := \frac{\varepsilon}{\varepsilon - 1} W(t) A_i(t)$$

thus, its logarithm $p_i^*(t) := \log P_i^*(t)$ evolves as

$$dp_i^*(t) = \mu dt + \sigma dB_i(t) \tag{6}$$

and I impose the initial condition $p_i^*(0) \sim \mathcal{N}(p_{i,0}, z_{i,0})$ with an arbitrary value (p_0, z_0).

3.2.1 Information Structure and Filtering Problem

Firms' information structure is based on Afrouzi and Yang (2021), and hereafter, I drop the subscript i for brevity. I assume that firms cannot directly observe their idiosyncratic productivity $a(t)$ but can acquire information about it from noisy signals. By observing the signals, firms estimate their productivity $a(t)$, or equivalently, their optimal price $P^*(t) := \frac{\varepsilon}{\varepsilon-1} W(t) A(t)$ and make decisions. I define the signal process $s(t)$ as

$$ds(t) = p^*(t)dt + \sigma_s(t)dV(t) \tag{7}$$

where $V(t)$ is a standard Brownian motion, independent of $p^*(t)$. The volatility $\sigma_s(t)$ represents the accuracy of the signal. Intuitively, as $\sigma_s(t)$ gets lower, a signal has more accurate information on $p^*(t)$. Note that logarithmic form of the optimal price $p^*(t)$ enters as the drift of the signal so as to make the filtering problem tractable[5].

Firms form their beliefs using the set of all previous signals at any moment. Thus, given the path of $\{\sigma_s(t), t \geq 0\}$, the information set at time t is defined as the σ-algebra generated by the history of the signals $s(t)$:

$$I(t) = \sigma\{s(r); r \leq t\} \tag{8}$$

Given this information set, firms form their beliefs about $p^*(t)$. Let $\hat{p}(t) = \mathbb{E}[p^*(t)|I(t)]$ be the best estimate of the optimal price and let $z(t) = \mathbb{E}[(p^*(t) - \hat{p}(t))^2|I(t)]$ be its variance. Notice that, from now on, I refer to $z(t)$ as the firm level uncertainty, following Baley and Blanco (2019). Proposition 1 establishes the law of motion for optimal price estimates and

[5] I consider that the firm receives signals about its optimal price, rather than that about its productivity, but the same results can be derived even if I define the signal process $s(t)$ as $ds(t) = a(t)dt + \sigma_s(t)dV(t)$.

uncertainty.

Proposition 1 *Given a sequence of accuracy of signals $\{\sigma_s(t), t \geq 0\}$ and the information set defined as (8), the firm's belief about $p^*(t)$ conditional on $I(t)$ is Gaussian $p^*(t)|I(t) \sim \mathcal{N}(\hat{p}(t), z(t))$, where $(\hat{p}(t), z(t))$ evolve as follows:*

$$d\hat{p}(t) = \mu dt + \frac{z(t)}{\sigma_s(t)} d\hat{B}, \qquad \hat{p}(0) = p_0 \tag{9}$$

$$dz(t) = \left(\sigma^2 - \frac{z(t)^2}{\sigma_s(t)^2}\right) dt, \qquad z(0) = \sigma_0 \tag{10}$$

where $\hat{B}(t)$ is given by $d\hat{B}(t) = \frac{1}{\sigma_s(t)}(ds(t) - \hat{p}(t)dt)$ and it is a standard Brownian motion under the information set.

Proof. See, for example, Theorem 6.2.8 of Øksendal (2013).

Proposition 1 shows that the firm's estimated optimal price and its variance, or the firm's uncertainty, depends on the accuracy of signals $\sigma_s(t)$. When the signal is accurate (low $\sigma_s(t)$), it is optimal for the firm to update its estimate $\hat{p}(t)$ using new information from the signal and its uncertainty $z(t)$ decreases. On the other hand, when the firm receives noisy signals (high $\sigma_s(t)$), it puts high weight on the old information from the previous estimates and the associated uncertainty increases.

3.2.2 Information Choice and Cost Function

I assume that firms decide how attentive to signals by means of choosing the path of signals' precision $\{\sigma_s(t) \in \mathbb{R}_+ \cup \{\infty\}, t \geq 0\}$ subject to the information cost which depends on the amount of acquired information from signals. Hence, firms become rationally inattentive due to the existence of the information cost.

Before specifying the information cost function, it is convenient to introduce a variable $\eta(t) := z(t)/\sigma_s(t)^2$ that is the *Kalman-Bucy gain* of the above filtering problem. Then, because the deterministic Riccati equation (10) with the initial condition indicates that $z(t)$ is predetermined at time t by the past choices, a choice of $\sigma_s(t) \geq 0$ is equivalent to a choice of $\eta(t) := z(t)/\sigma_s(t)^2 \geq 0$. With this notation, the evolution of $\{\hat{p}(t), z(t)\}$ can be written as:

$$d\hat{p}(t) = \mu dt + \sqrt{z(t)\eta(t)} d\hat{B}(t) \tag{11}$$

$$dz(t) = (\sigma^2 - z(t)\eta(t))dt \tag{12}$$

The *Kalman-Bucy gain* $\eta(t)$ represents the relative weight of new information from the signals to old information in the estimation process (11),(12). At all times, the firm decides how much attention it pays to the signals by picking a sequence of $\{\eta(t), t \geq 0\}$, where a higher value of $\eta(t)$ means that the firm decides to obtain and process new information on its optimal price from the signals. Moreover, the accuracy of signals affects the value of $\eta(t)$. When a signal is accurate (low $\sigma_s(t)$), the firm gives importance to the signal (high $\eta(t)$) and updates its estimates by processing information from the signal.

Here, I impose an exogenous restriction on firms' information choice behavior. Specifically, I restrict the domain of $\eta(t)$ so that $\eta(t)$ is at least larger than or equal to a non-zero lower bound $\bar{\eta} > 0$ at every moment: $\eta(t) \in [\bar{\eta}, \infty], \forall t \geq 0$. Intuitively, this assumption means that every firms are paying at least some attention to signals. Without this exogenous lower bound, there exists a stationary equilibrium in which some firms completely ignore the signals and they have never changed their estimates and their nominal price. To eliminate such an unrealistic case, I introduce the exogenous lower bound of $\eta(t)$.

Turn now to the specification of the costs of processing information. I assume that the information cost depends on the amount of acquired information about the optimal price $p^*(t)$ from observing the signal $s(t)$. Following the seminal work by Sims (2003), the amount of information obtained about one random variable by observing the other random variable is measured by the (time limit of) *mutual information* between the two random variables[6]. I follow the definition in Boyarchenko (2012).

Definition 1 *Let x and y be two random processes on the real line, and denote by $\mu_x, \mu_y, \mu_{x,y}$ the probability measures induced on the canonical space $(\Omega, \mathcal{G})$ by x, y and (x, y) respectively. Then, the mutual information between processes x and y over the time interval $[0, t]$ is*

$$\mathcal{I}_t(x, y) := \mathbb{E}\left[\log \frac{d\mu^t_{x,y}}{d[\mu^t_x \times \mu^t_y]}(x, y)\right]$$

where μ^t is the restriction to $\mathcal{G}_t$ and $\mu^t_x \times \mu^t_y$ is the Cartesian product of the corresponding measures.

According to definition 1, I assume that the information cost is specified as a convex function of the time limit of the mutual information between the process of signal s and that of optimal price p^*. Given the Gaussian structure of my framework, I can express the mutual information succinctly. The following proposition gives a concrete illustration of it.

[6] Canonical rational inattention paper such as Sims (2003) imposes the capacity constraint, or a fixed upper bound on the amount of imformation. On the other hand, this paper assumes that the firm faces a labor cost when it processes information from the signal. It allows me to generate state-dependent information acquisition behavior.

Proposition 2 *If two random processes, (p^*, s) evolves as (6), (7) and the following conditions are satisfied:*

1. *Equation (7) has a strong solution.*
2. $\int_0^t \mathbb{E}\left[p^*(s)^2\right] ds < \infty$.

Then, the mutual information between s and p^* is expressed as

$$\mathcal{I}_t(p^*, s) = \frac{1}{2}\int_0^t \mathbb{E}\left[\frac{(p^*(r) - \hat{p}(r))^2}{\sigma_s(r)^2} \;\middle|\; I(t)\right] dr$$

Proof. See Theorem 16.3 of Liptser and Shiryaev (2013).

By using the result in Proposition 2, the time limit of mutual information is given by

$$\frac{d\mathcal{I}_t(p^*, s)}{dt} = \frac{1}{2\sigma_s(t)^2}\mathbb{E}\left[(p^*(t) - \hat{p}(t))^2 | I(t)\right] = \frac{1}{2}\frac{z(t)}{\sigma_s^2(t)} = \frac{1}{2}\eta(t) \tag{13}$$

Thus, the amount of information encoded in the signal is equal to the *Kalman-Bucy gain* $\eta(t)$. In this paper, I assume that if the firm processes information, it has to pay the information cost equal to c_{info} untis of labor, and c_{info} is a quadratic function of $\eta(t)$[7]:

$$c_{info} := \frac{\chi}{2}\left(\eta(t) - \bar{\eta}\right)^2 \tag{14}$$

where χ is a information cost parameter. With this specification, firms do not have to pay the information cost when they choose the least level of attention $\bar{\eta}$. Notice that, since the cost function is assumed to be quadratic in attention, the firm is never able to observe the true value of its optimal price. This contrasts with the observation cost model such as Alvarez et al. (2011), in which once firms pay an observation cost, they can see their true optimal price level.

3.2.3 Firm's Optimization Problem

Suppose that the firm makes decisions based on its estimates $(\hat{p}(t), z(t))$. Let $x(t)$ be the firm's perceived price gap between its own nominal price $P(t)$ and estimated optimal price $\hat{p}(t)$, which is defined as:

$$x(t) := \log P(t) - \hat{p}(t)$$

[7] As here, Lei (2019) and Andrei and Hasler (2014) consider quadratic cost functions in the different context.

Then, while the firm maintains its price, it evolves according to

$$dx(t) = -\mu dt + \sqrt{z(t)\eta(t)}d\hat{B}(t) \tag{15}$$

With this notation, the firm's *perceived* instantaneous profit is written as a function of t and x:

$$\mathbb{E}\left[\hat{\Pi}(t)\middle|I(t)\right] = W(t)f(t,x) := W(t)\alpha^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} c(t,x)^{1-\gamma\varepsilon}e^{-\varepsilon x}\left[e^{x} - \frac{\varepsilon-1}{\varepsilon}\right]$$

and the aggregate consumption is given by

$$c(t,x) = \left(\frac{\varepsilon-1}{\alpha\varepsilon}\right)^{\frac{1}{\gamma}}\left[\int e^{(1-\varepsilon)x}\phi_t(x)dx\right]^{\frac{1}{\gamma(\varepsilon-1)}}$$

where $\phi_t(x)$ is the marginal density of x at time t.

Given the state variables $(x(t), z(t))$, the firm's behavior can be described as the following combined stochastic and impulse controls. First, the firm's information acquisition behavior is represented by a path of attention at every moment $\{\eta(t); t \geq 0\}$. Second, the firm's price adjustment behavior is described by an impulse control, or a sequence of the price adjustment times $0 \leq \tau_1 \leq \tau_2 \leq \cdots$ and corresponding price adjustment sizes $\xi_1, \xi_2, \cdots; \forall j, \xi_j \in \Xi \subseteq \mathbb{R}$ at the time.

Notice that the firm maximizes the expected sum of its profits, discounted at $Q(t)$ and from the household's optimization conditions, I can derive $Q(t)W(t) = Q(0)W(0)e^{-\rho t}$. Hence, given an initial condition p_0^*, the state's law of motion (12),(15), the filteration $\{I(t)\}_{t\geq 0}$, and arbitrary controls $\theta := \{\eta(t)\}_{t\geq 0}, \{\tau_j, \xi_j\}_{j=1}^{\infty}$, the firm's objective function is described by

$$J(0, x(0), z(0); \theta) := \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}[f(t,x) - \frac{\chi}{2}(\eta(t) - \bar{\eta})^2]dt - \kappa \sum_{0\leq\tau_j\leq\infty} e^{-\rho\tau_j}\right]$$

Thus, the firm's optimization problem is finding the value function and the corresponding optimal stochastic and impulse controls θ^* such that

$$V(t,x,z) = \sup_{\theta} J(t,x,z;\theta)$$

To solve this problem, I use an equivalent partial deffienrential equation formulation[8] that

[8]See chapter 9 of Øksendal and Sulem (2019) and Seydel (2009) for details.

can be solved via numerical methods. Specifically, with some technical conditions, the value function $V(t,x,z)$ is given by the solution to the following Hamilton-Jacobi-Bellman quasi-variational inequality (HJBQVI)[9]:

$$\max\left\{V_t - \rho V - \mu V_x + \sigma^2 V_z + f(t,x) + \sup_{\eta \geq \bar{\eta}}\left\{\left(\frac{1}{2}V_{xx} - V_z\right) z\eta - \frac{\chi}{2}(\eta - \bar{\eta})^2\right\}\ ,\ \mathcal{M}V - V\right\} = 0 \tag{16}$$

where V_x is a partial derivative of V with respect to x, and $\mathcal{M}$ is the intervention operator that returns the optimal value resulting from an intervention. Let Γ be the function that determines the size and direction of intervention, then the definition of $\mathcal{M}$ is given by

$$\mathcal{M}V(t,x,z) = \sup_{\xi}\left\{V\left(t, \Gamma(t,x,\xi), z\right) - \kappa\ ;\ \xi \in \Xi\right\} \tag{17}$$

Notice that the second term of the left hand side represents the cost or profit of making an intervention, and in this context, that is corresponding to the menu cost.

The firm's optimal behavior is characterized by an inaction region $\mathcal{D}$ on which the firm leaves its price unchanged, optimal return point $\hat{x}(t,z)$ which is the price gap the firm chooses in the price adjustment, and the intensity of attention to information $\eta(t,x,z)$. Let $\underline{x}(z)$ and $\bar{x}(z)$ be the lower and upper bound of the inaction region at time t so that the inaction region is written as the set $\mathcal{D}_t = \{(x,z) : x \in [\underline{x}(z), \bar{x}(z)]\}$. It is worth noting that, because the firm's state space is two-dimensional, its policy functions $\{\mathcal{D}(x,z), \hat{x}(z), \eta(x,z)\}$ are also two-dimensional. It implies that the firm's price adjustment and information acquisition decisions are influenced by both price gap and uncertainty. Thus, it can be seen that the two behaviors —price adjustment and information acquisition—are strongly interconnected through the state variables and determined by each other.

Lastly, the distribution of state variables at time t; $\phi_t(x,z)$ is determined by the Fokker-Planck equation with the effect of price adjustment. Without price adjustment, the Fokker-Planck equation is written as

$$\frac{\partial \phi}{\partial t} = (\mu + z\eta_x)\phi_x + (z\eta - \sigma^2)\phi_z + \frac{z\eta}{2}\phi_{xx} + \left(\eta + z\eta_z + \frac{z\eta_{xx}}{2}\right)\phi \tag{18}$$

This equation represents the time evolution of the joint density function within the inaction region. In addition, the density at the threshold of the inaction region jumps to the optimal return point due to the price adjustment. In this paper, I numerically implement this proce-

[9]Here, I implicitly consider the discounted HJBQVI that is given by multiplying by $e^{\rho t}$ and abuse the notation V

dure by using the intervention matrix which is the natural discretization of the intervention operator $\mathcal{M}$ defined in (17).

4 Stationary Equilibrium

In this section, I analyze the equilibrium properties of my framework. I focus on a stationary equilibrium at which the growth rate of money supply is constant at μ. Then, the stationary equilibrium is defined as follows.

Definition 2 *Given the exogenous stochastic processes of idiosyncratic productivity $B_i(t)$, and idiosyncratic observation noise $V_i(t)$, a stationary equilibrium is a set of stochastic processes that consists of an allocation of the representative household, $\{c(t), l(t), M(t)\}_{t\geq 0}$, a value function V and a set of policy functions $\{\underline{x}(z), \hat{x}(z), \bar{x}(z), \eta(x,z)\}$ for every firm $i \in [0,1]$, a set of prices $\{P(t), W(t), r(t), Q(t)\}$, and a stationary distribution over firm states $\phi(x,z)$ such that*

1. *the representative household maximize her utility function,*
2. *the value function V solves the HJBQVI (16) with $V_t = 0$ and $\{\underline{x}(z), \hat{x}(z), \bar{x}(z), \eta(x,z)\}$ are associated policy functions,*
3. *markets clear at each date, and*
4. *the stationary distribution is consistent with actions.*

Notice that the money growth rate μ pins down the equilibrium inflation rate and affects the firms' behavior.

4.1 Parameter Calibration

Before providing numerical analyses, I set the parameter values. For many of them, I follow Golosov and Lucas (2007). The discount rate is 0.04 to match an annual risk-free rate of 4 percent. The value of the substitution of elasticity $\varepsilon = 7$ implies that the firm's markup is about 16 percent. The disutility of labor $\alpha = 6$ implies that 37 percent of unit of time is allocated to work.

The remaining parameters are the information cost χ and the exogenous lower bound of attention $\bar{\eta}$. I assume that $\chi = 6$ so that the average sum of Kalman-Bucy gain matches the estimate of 0.50 in Coibion and Gorodnichenko (2015). Finally, the exogenous lower bound is set to $\sigma^2/2$ so that the most inattentive firm acquire only 1 percent of estimated Kalman

parameter	Description	Value	Source
ρ	Discount rate	0.04	Golosov and Lucas (2007)
α	Disutility of labor	6	Golosov and Lucas (2007)
γ	Relative risk aversion	2	Golosov and Lucas (2007)
ε	Elasticity of substitution between goods	7	Golosov and Lucas (2007)
σ	Standard deviation of idiosyncratic shocks	$(0.011)^{\frac{1}{2}}$	Golosov and Lucas (2007)
κ	Menu cost	0.0025	Golosov and Lucas (2007)

Table 1: Parameter Calibration

gain. Of course, these assumptions are very simple and I will estimate these parameter values to match the micro firm data in the future research. Notice, however, that every theoretical findings in this chapter hold within the realistic range of values of the two parameter χ and $\bar{\eta}$.

4.2 Numerical Analysis

In this subsection, I describe the numerical results of the stationary equilibria with different levels of trend inflation rates and provide some qualitative findings and predictions.

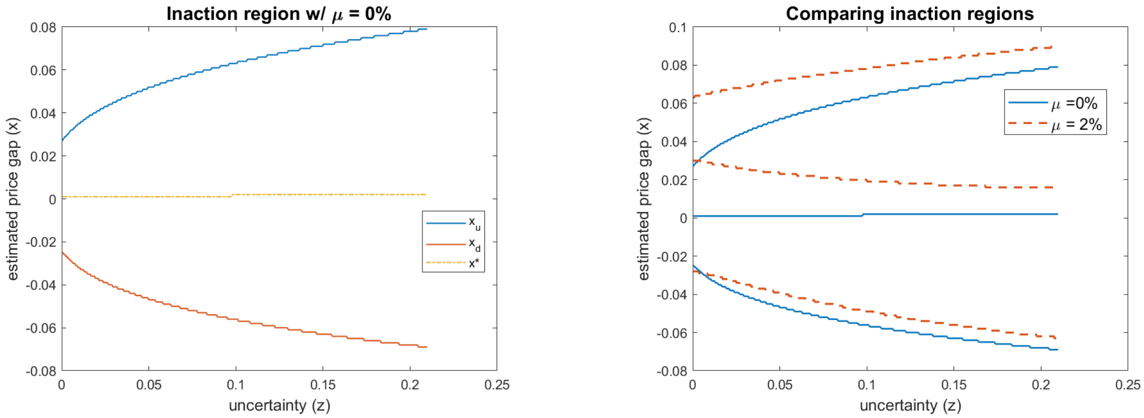
4.2.1 Firms' Decision Rules

Inaction region Firstly, I describe the firm's price adjustment decision. As I said, it is characterized by the inaction region and the optimal return point, $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$. One notable difference from the basic menu cost model is that the price adjustment decision depends on not only the price gap but uncertainty. Figure 1 shows the equilibrium inaction region and return points. Panel (a) is the triplet $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$ under the zero inflation rate case and panel (b) is the comparison of the triplet between zero and two percent inflation rate cases.

In both cases, the uncertainty widens the inaction region, which is due to the so-called *option value effect* proposed in, for example, Dixit (1991). As a result, the size of price change varies across firms according to their uncertainty. This cross-sectional variation in the price setting is consistent with the finding of Baley and Blanco (2019)[10].

Furthermore, I compute the inaction region with different trend inflation rates. Panel (b) shows that as the trend inflation rate increases, the triplet shifts upward for any value of uncertainty. Firstly, the optimal return point $\hat{x}$ becomes positive, while it is near zero

[10]They generate the cross-sectional variation by combining the information friction and fat-tailed shocks to idiosyncratic productivity. In this paper, instead of the fat-tailed shock, I assume that firms are able to change their attention to the signals by paying information costs. Since this decision depends on both x and z, the uncertainty does not converge to a constant according to the deterministic Riccati equation (12).



(a) 0% trend inflation **(b)** comparing 0% and 2% trend inflation

Figure 1: Inaction Region

Notes. In panel (b), blue lines are the equilibrium triplet $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$ under the zero inflation rate and red dotted lines are that under the two percent inflation rate.

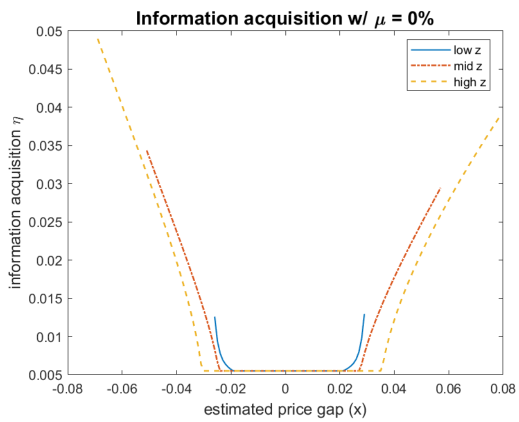
under zero inflation rate. This means that when the trend inflation rate is zero, it is optimal for firms to revise their price to equal the estimated optimal one. However, when the trend inflation rate is positive, firms choose the positive estimated price gap, that is, they change the price to some level higher than the estimated optimal value. This is because, if the trend inflation is positive, firms expect a higher optimal price in the future. As panel (b) shows, this mechanism weakens as uncertainty increases, because under high uncertainty, firms receive more volatile signals and the impact of inflation is reduced.

Along with this change in $\hat{x}$, the inaction region also shifts upward. Notice, however, that there is a difference in the range of change between the upper and lower bounds of the inaction region. The reason is that because of the positive inflation rate, the firm has more opportunities to raise its price, and thus it tries to control the frequency of price increases to save menu costs. It brings the lower bound down. Hence, the impact of the trend inflation rate is asymmetric between the upper and lower bounds.

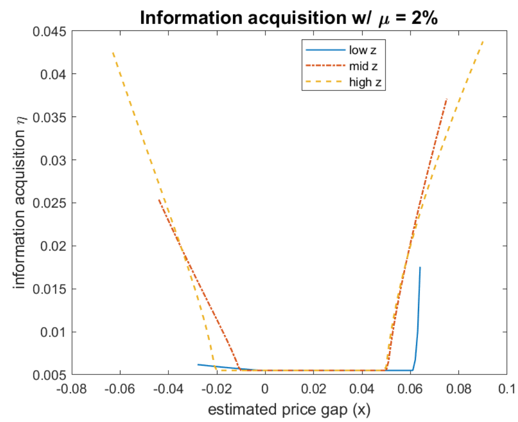
Intensity of attention Next, I describe the firm's information acquisition decision. The intensity of attention to the signal and the amount of acquired information from the signal are both represented by the optimal $\eta(x, z)$, which is given by the following maximization problem in the HJBQVI (16):

$$\sup_{\eta \geq \bar{\eta}} \left(\frac{1}{2} V_{xx} - V_z \right) z\eta - \frac{\chi}{2}(\eta - \bar{\eta})^2$$

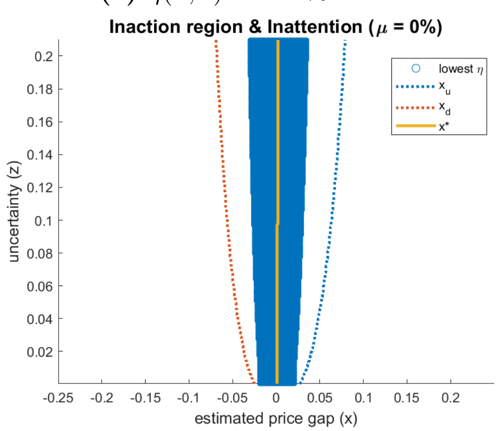
The coefficient of first term $\left(\frac{1}{2}V_{xx} - V_z\right) z$ represents the marginal benefit of paying additional attention to the signal and the second term is the cost of attention or information acquisition. The marginal benefit tells us about two channels through which information acquisition contributes to the firm's profit. First, V_{xx} stands for the speed of increase or decrease rate of the firm's value in terms of the price gap. Thus, intuitively, if the firm's value is sensitive to changes in the price gap (positive and high V_{xx}), perceiving the more accurate price position by paying additional attention becomes beneficial. Second, $-V_z$ represents the reduction rate of loss due to a marginal decrease in uncertainty. It means that acquiring information increases the firm's value via mitigating the loss caused by uncertainty. Notice that both channels are scaled by z. This implies that firms with relatively high uncertainty benefit more from information acquisition.



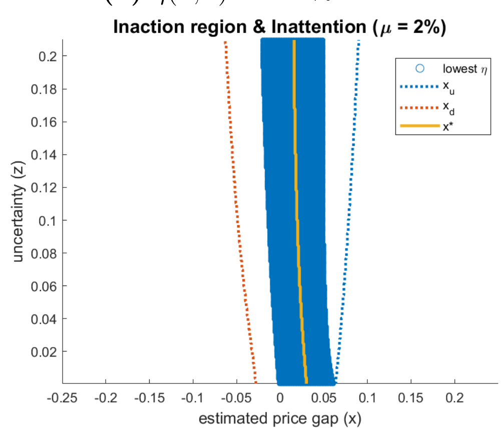
(a) $\eta(x, z)$ with 0% inflation



(b) $\eta(x, z)$ with 2% inflation



(c) Inaction & Inattention Region with 0% inflation



(d) Inaction & Inattention Region with 2% inflation

Figure 2: Information Acquisition

Notes. In the lower panels, blue-colored area represents the pair of states (x, z) at which the optimal information acquisition is equal to the exogenous lower bound: $\eta(x, z) = \bar{\eta}$.

The upper panels of figure 2 illustrate the equilibrium $\eta(x, z)$ with different levels of trend inflation rate and the lower panels plot the area where the firm chooses the lowest value of η and the price setting triplet $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$. Regardless of the value of inflation, the firm has no incentive to acquire costly information around its optimal return point $\hat{x}$ but as the estimated distance between its own price and the optimal one opens up, the firm is willing to pay the cost and acquire information. Its acquisition volume is at its maximum in the vicinity of the threshold of the inaction region $\{\underline{x}(z), \bar{x}(z)\}$. Indeed, V_{xx} takes small or negative value around $\hat{x}(z)$ but as x moves away from the optimal point, the value of V_{xx} increases. In addition, panel (a) and (b) show that the amount of acquired information before price adjustment is greater for more uncertain firms[11].

Moreover, information acquisition depends on the level of trend inflation[12]. Panel (b) shows that, when the trend inflation is positive, firms try to acquire more information in advance when they lower prices than when they raise them. This is because, if there is no idiosyncratic shock, the price gap is expected to fall through time due to positive inflation. Thus, when the firm estimates the positive price gap, it has a strong incentive to acquire new information about idiosyncratic shocks.

The results of firm's policy functions show that there is a strong interaction between information acquisition and price setting behavior. Firms are inattentive while they perceives their own price to be close to the optimal one, but as it moves away, they try to acquire information and finally changes their price after acquiring enough accurate information. Furthermore, This interaction is affected by the level of inflation rate. With a positive inflation, firms do not try to acquire as much information in advance when raising the price as when lowering it.

Expected price change time I show that the firm's behavior depends on the level of its uncertainty. Specifically, high uncertainty widens the inaction region and increases the amount of information acquisition. The former delays the price adjustment, but the latter increases the estimate volatility of x and raises the probability of hitting the boundaries of the inaction region. Then, which force dominates and whether does uncertainty increase or decrease the price adjustment frequency? To answer this question, I derive the expected duration between two price adjustments conditional on the state. Let $T(x, z) := \mathbb{E}[\tau | x, z]$ be

[11] The similar results are shown independently in Yang (2020). While the shape of η in my model depends largely on the convex or quadratic information cost function, he assumes that the cost is linear in Shannon's mutual information and generates the similar shape of information acquisition.

[12] Notice that even if the inflation rate is zero, information acquisition will not be symmetrical. It is mainly due to asymmetricity of the instantaneous profit function $\hat{\Pi}$. Although $\hat{\Pi}$ is a concave function of x and it takes its maximum value at $x = 0$ as with the second-order approximated profit function $\tilde{\Pi} := -const \times x^2$ which is used in, for example, Alvarez and Lippi (2014), Baley and Blanco (2019), but $\hat{\Pi}$ takes a larger value when x is positive: $\hat{\Pi}(x) > \hat{\Pi}(-x)$. Thus, the resulting value function is also asymmetric and the inaction region is wider in the positive region. As a result, V_{xx} (and η) takes a larger value around $\underline{x}$ than $\bar{x}$.

the equilibrium expected time for the next price adjustment given the current state. Then, the function $T(x, z)$ satisfies the following HJBQVI with the same associated policy functions with the original firm's problem (16): $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$ and $\eta(x, z)$:

$$\max\left\{ 1 - \mu T_x + (\sigma^2 - z\eta(x,z))T_z + \frac{z\eta(x,z)}{2}T_{xx}\ ,\ \ -T \right\} = 0$$

Notice that the intervention operator $\mathcal{M}$ is defined as $\mathcal{M}T = 0$ since the time is reset to 0 by the price adjustment.

Figure 3 illustrates the expected time to reach the boundary of the inaction region from the optimal return point $\hat{x}(z)$ conditional on the uncertainty $\mathbb{E}[\tau|\hat{x}(z), z]$ with different trend inflation rates.

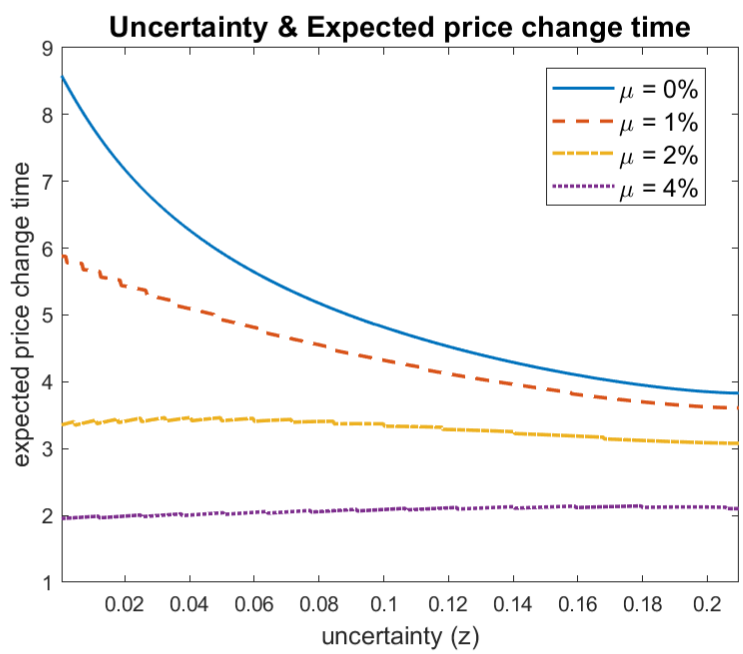


Figure 3: Expected time for the next price change

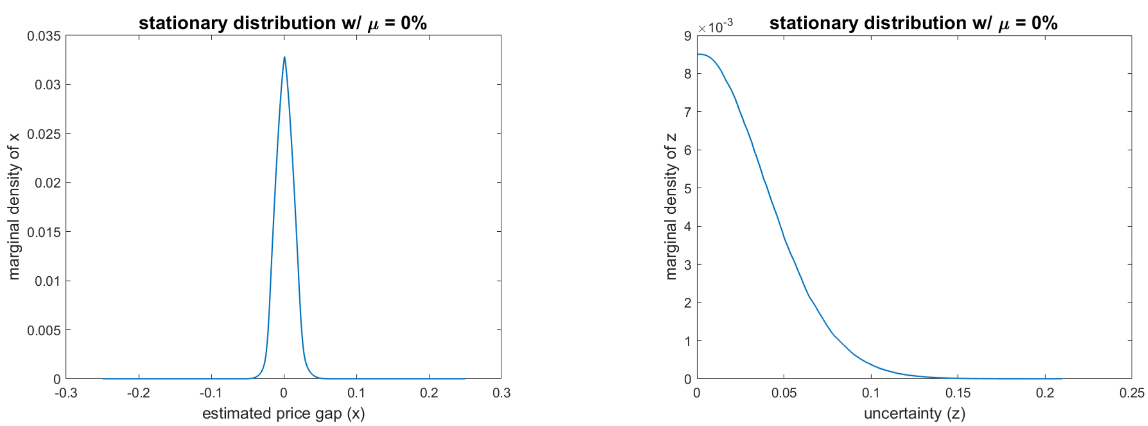
The figure indicates that under the low inflation rate, the effect of heightening the estimate volatility dominates that of widening the inaction region. Intuitively, more uncertain firms adjust prices based on incorrect estimates of the optimal price. Then, the probability that the revised price is far from the actual value of optimal price gets higher. It shortens the expected time for the next price change. This result is the same as Baley and Blanco (2019), who show that there is a negative relation between uncertainty and expected adjustment time when the menu cost and signal's volatility σ_s are small. They assume that the inflation rate is zero and the firm's intensity of attention is fixed. I show that this negative relationship still holds under low inflation rates, even if firms can choose the intensity of attention depending on its state.

A novel part of my analysis is comparing the expected price change time for different trend inflation rates. Firstly, I show that the conditional expected time until the next price adjustment is shortened according to the inflation rate for any value of z. It is obvious that a positive inflation rate reduces the time to reach the lower bound of the inaction region and increases the time to reach the upper bound. Moreover, I have already shown that the high inflation rate widens the inaction region and lowers η, or the volatility of x around the lower bound of the inaction region $\underline{x}(z)$. Thus, the finding suggests that the latter change amplifies the effect of the positive inflation rate on the expected time to reach the lower bound, which in turn dominates the effect of widening the inaction region.

Furthermore, I show that when the inflation rate is high, the uncertainty is less likely to reduce the expected time, or rather slightly increase it. The reason is that the high inflation rate reduces the amount of η around the lower bound of the inaction region. It undermines the volatility effect of uncertainty. Intuitively, firms tend to focus on aggregate variables rather than idiosyncratic shocks when they raise prices, and thus, uncertainty on idiosyncratic conditions has few effects on their price setting decisions.

4.2.2 Stationary Distribution and Aggregation

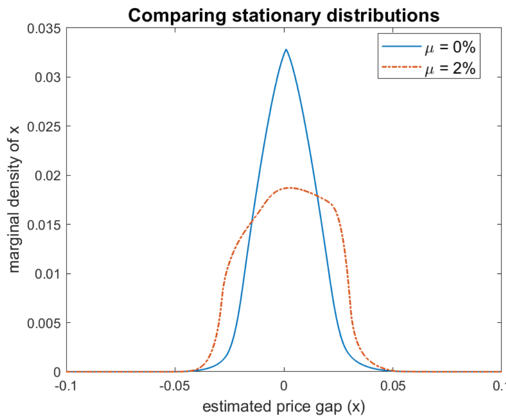
Stationary distribution Figure 4 illustrates the marginal stationary distribution of the firm's state under the zero inflation rate. Panel (a) shows that the estimated price gap x has a leptokurtic distribution. It means that many of the firms perceive that their prices are close to the optimal value in which firms have no incentive to obtain costly information.



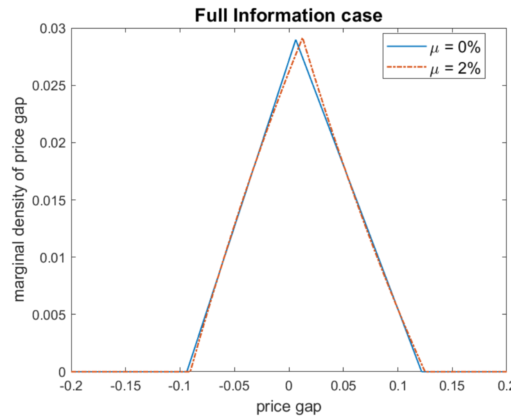
(a) Marginal distribution of x **(b)** Marginal distribution of z

Figure 4: Stationary distribution with zero inflation rate

The kurtosis of the estimated price gap distribution depends on the value of the trend inflation rate. Figure 5 compares the marginal distribution of the (estimated) price gap in both zero and two percent inflation rate cases. Panel (a) shows that with a positive inflation rate, the density around the return point $\hat{x}$ is reduced and the density around the threshold of the inaction region is increased instead. It implies that as the inflation rate raises, the extensive margin of the price setting, or the fraction of firms who change their price increases.



(a) Imperfect Information



(b) Full Information

Figure 5: Comparing marginal distribution of x

Notes. Panel (a) is the stationary marginal distributions of the estimated price gap in the original rational inattention model. On the other hand, panel (b) plot the stationary distribution of the actual price gap in the full information menu cost model. In both panels, blue line shows the distribution under zero inflation rate and red dotted line shows that under two percent inflation rate.

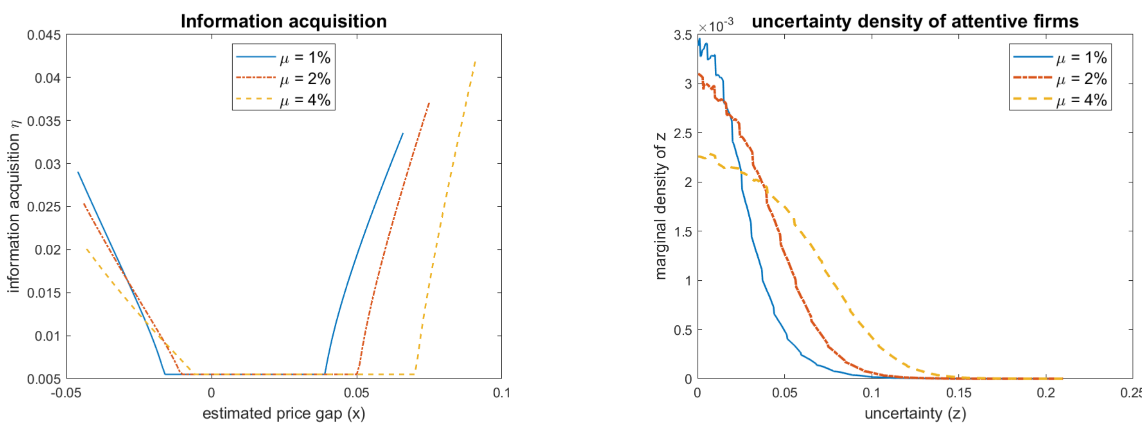
These differences in the price gap distribution caused by the value of trend inflation rate are not observed in the full information menu cost model such as Golosov and Lucas (2007). I replicate the standard menu cost model of Golosov and Lucas (2007) and plot the stationary distribution of the actual price gap with different inflation rates as shown in panel (b). The figure shows that, as the trend inflation rate increases, the inaction region and the optimal return point slightly shift right, but the kurtosis of the distribution is almost the same. Hence, regardless of the value of trend inflation, the number of firms who change their price at each date does not change much.

My theoretical prediction about the relation between the extensive margin of price setting and the value of trend inflation rate is empirically plausible in the sense that it is consistent with recent empirical findings documented by Watanabe and Watanabe (2018). They find that, when the inflation rate is close to zero, the share of items whose price stays unchanged increases as the trend inflation rate approaches zero.

Moreover, my model offers the mechanisms underlying their findings. When the trend inflation rate is positive, the firm can anticipate that, after some time, its own price is likely to be less than the optimal value due to the positive inflation rate. Then, the firm has an incentive to acquire costly information and prepare the price change. On the other hand, when the trend inflation rate is zero, there is no aggregate drift that gives some information about the price gap to the firm. To understand this, consider an extreme example: when the exogenous lower bound of η is equal to 0, in the vicinity of optimal return point $\hat{x}$, firms optimally choose to be *completely inattentive*. It means that the firm who perceives that its price is equal or close to the optimal one has no incentive to update its belief. As a result, the stationary distribution of x becomes a Dirac at the point $\hat{x} = 0$. All firms believe their price to be optimal and there is no price adjustment at all.

Hence, the result of my model predicts that as the trend inflation rate gets closer to zero, the fraction of inattentive firms increases and it reduces the extensive margin of price change. Next, I examine the relation between the value of trend inflation rate and the distribution of firm uncertainty.

Average uncertainty There are two underlying mechanisms through which inflation rate affects firm uncertainty. First, the level of trend inflation rate influences the intensive margin of information acquisition, or the amount of information acquired before a price change per firm. In panel (a) of figure 6, I compare the amount of information acquisition $\eta(x, z)$ with



(a) Comparing $\eta(x, z)$ **(b)** Comparing ϕ_z conditional on $\eta > \bar{\eta}$

Figure 6: Information acquisition and firm uncertainty

Notes. Panel (a) plots the equilibrium $\eta(x, z)$ for an arbitrary value of z. Panel (b) illustrates the marginal density of z conditional on $\eta(x, z) > \bar{\eta}$. It is interpreted as the uncertainty distribution of attentive firms. In both panels, blue line is for 1%, red-dashed line is for 2%, and yellow-dotted line is for 4% inflation rate.

fixed z for different inflation rates: $1, 2, 4\%$. The figure shows that a high inflation rate lowers the optimal amount of information acquired when raising prices and raises the amount when lowering them. Since the fraction of firms who raise their prices increases with the inflation rate, a high inflation rate increases uncertainty of the "attentive" firm, who pays some costs to acquire information. Panel (b) of figure 6 plots the density of uncertainty z for those attentive firms who choose $\eta > \bar{\eta}$. It shows that with higher inflation rates, attentive firms become more uncertain because firms have less incentive to pay attention to the idiosyncratic shocks before raising prices. Thus, through this mechanism, a higher inflation rate increases the average firm uncertainty.

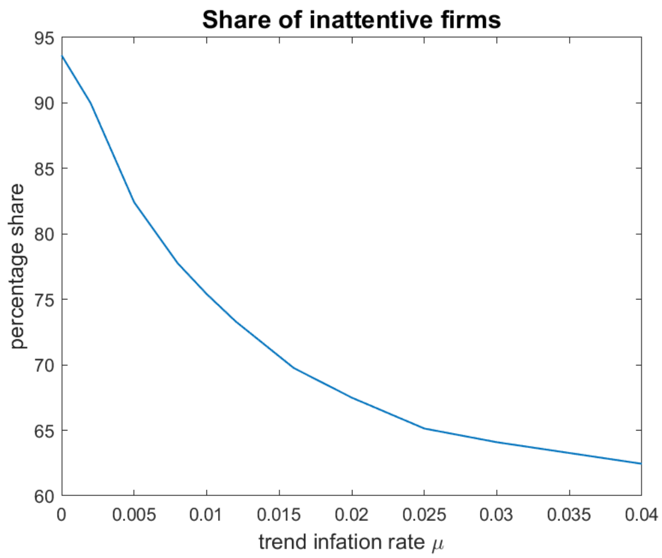


Figure 7: Share of inattentive firms

Notes. I plot the share of firms who choose the amount of information acquisition η equal to the lower bound $\bar{\eta}$ for each stationary equilibrium.

Second, the inflation rate also affects the extensive margin of information acquisition: an increase in the rate of trend inflation prompts more firms to acquire costly information. Figure 7 plots the share of "inattentive" firms, who have no incentive to pay the information cost and acquire only the lowest amount of information $\bar{\eta}$. Since the process of firm uncertainty is written as $dz = (\sigma^2 - \eta z)dt$, the uncertainty of inattentive firms continues to increase up to $\sigma^2/\bar{\eta}$[13]. Thus, *ceteris paribus*, a large share of inattentive firms raises the average firm uncertainty. The figure shows that the share of inattentive firms decreases with the level of inflation rate.

Hence, the trend inflation rate affects the firm uncertainty through these opposing mechanisms. Figure 8 plots the relation between the trend inflation rate and the average firm uncertainty calculated by $\int z\phi_z dz$. The figure shows that there is a non-monotone relation between them: the average firm uncertainty increases under near zero and higher inflation

[13] In my numerical results, $\bar{\eta}z < \sigma^2$ holds for any value of z with positive density.

rates. Furthermore, the driving force for high uncertainty is different in the two environments. When the inflation rate is high, the first mechanism dominates: each firm raises its price with less information about idiosyncratic shocks. On the other hand, when the inflation rate is close to zero, the second mechanism dominates: the fraction of firms who are willing to pay the cost to acquire information decreases.

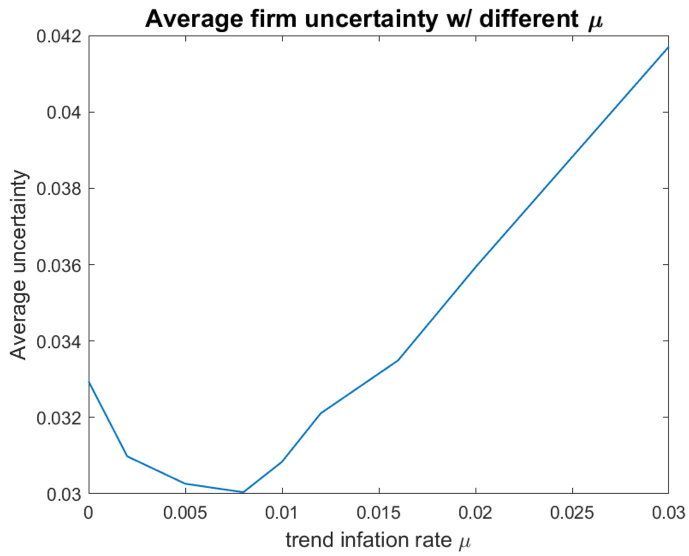


Figure 8: Average firm uncertainty

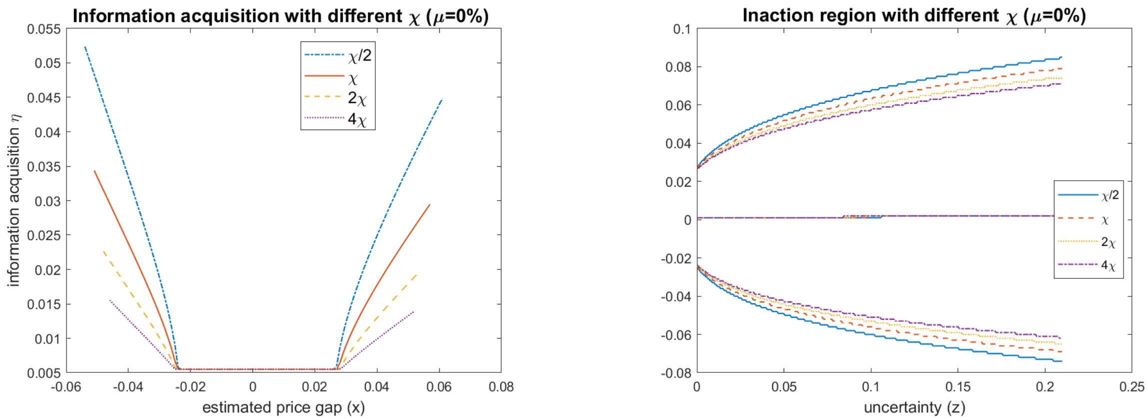
5 Comparative Statics

Now I consider how changes in the information cost parameter χ, the menu cost κ and the elasticity of substitution ε influence the firm's policy function and aggregate variables. Here, I only show the results under zero trend inflation rate, but the qualitative findings are the same regardless of the value of trend inflation.

5.1 Information Cost

First, I consider the effect of a change in χ on the firms' information acquisition and price setting behavior, where χ is a parameter in the information cost function: $c(\eta) := \frac{\chi}{2}(\eta - \bar{\eta})^2$.

The left panel of figure 9 illustrates the equilibrium information acquisition η with different value of χ. It shows that a higher value of χ reduces the optimal value of η for any x. It means that as the information cost gets expensive, the firm lowers its amount of information acquisition. Notice that the region where firms choose the lowest value of η does not change much with the value of χ. This implies that the value of information cost has a strong impact on the firm's decision of how much information to acquire, but it does not have much influence on the decision of when it starts obtaining costly information.

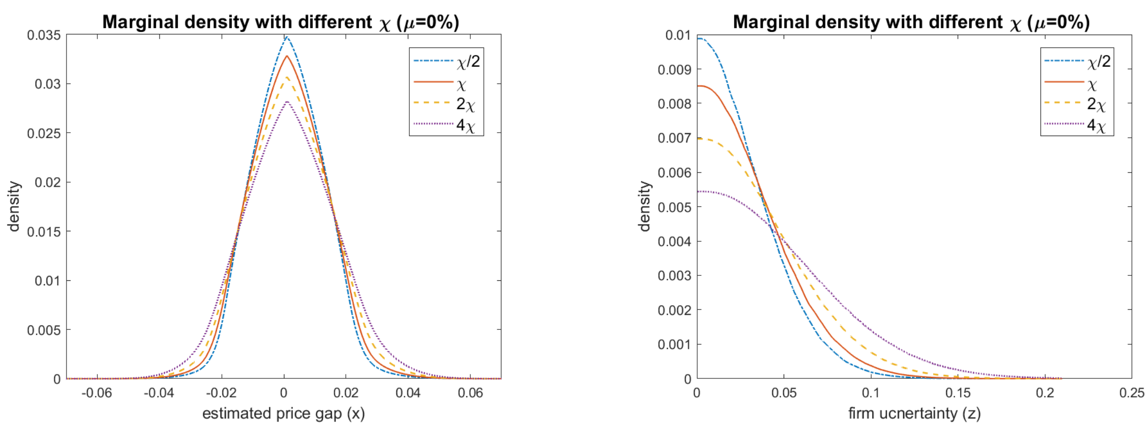


(a) Information acquisition **(b)** Inaction region

Figure 9: Firm's policy functions with different χ

Notes. The left panel shows the equilibrium value of $\eta(x, z)$ for a fixed z with different χ.

Interestingly, the change in the information cost also affects the firm's price adjustment decision. The right panel of figure 9 plots the inaction regions with different value of χ. The figure shows that the inaction region shrinks towards the optimal return price as χ gets larger. It implies that with higher information cost, the firm tends to adjust its price more quickly.



(a) Marginal density of x **(b)** Marginal density of z

Figure 10: Stationary distribution with different χ

Taken together, these results suggest that around the boundary of the inaction region, there is a substitution between price adjustment and information acquisition. Since both decisions incur costs, the firm prefers the cheaper one. In this case, rather than paying high information costs to make an accurate estimate, the firm pays the menu cost early to change its price.

Such a change in the firm's optimal decision affects the distribution among firms. Figure 10 illustrates the marginal distribution of the perceived price gap x and uncertainty z. The panel (a) shows that the distribution of price gap has fatter tail as χ increases. Combining this fact with the shrinkage of the inaction region implies that high information costs make firms adjust prices more frequently by a smaller amount. Since the amount of acquired information $\eta(x, z)$ decreases, firms get more uncertain as χ gets larger, shown in panel (b).

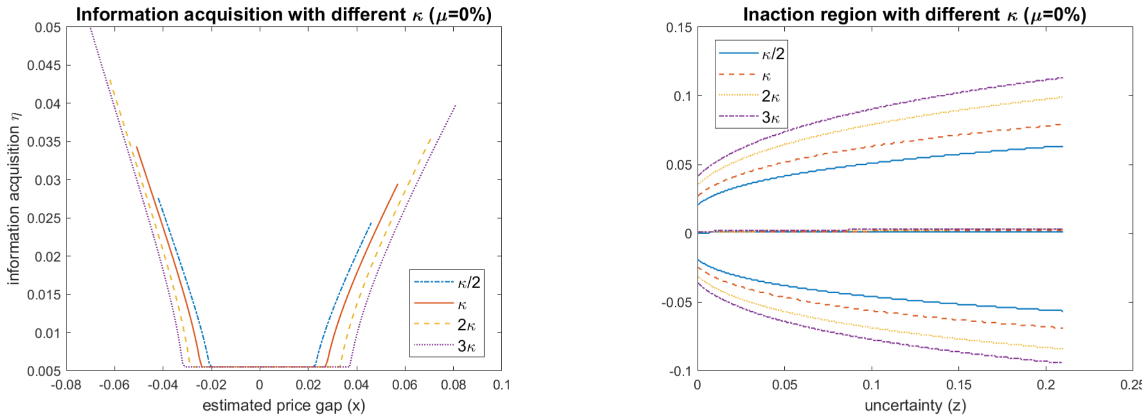
Therefore, the value of information cost parameter χ influences a firm's decision making, especially its behavior just before making a price adjustment. With a high χ, the firm gets inattentive to the signals in a sense that it reduces the amount of information acquired around the boundary of the inaction region. Furthermore, the firm tends to adjust its price more quickly instead of paying the information cost. As a result, as χ gets larger, the fraction of firms who pay the information cost and adjust price increases, but those firms do not acquire enough information before changing price. Hence, the firms' average uncertainty increases.

5.2 Menu Cost

Second, I consider the effect of a change in the menu cost κ on firms' behavior and their distribution.

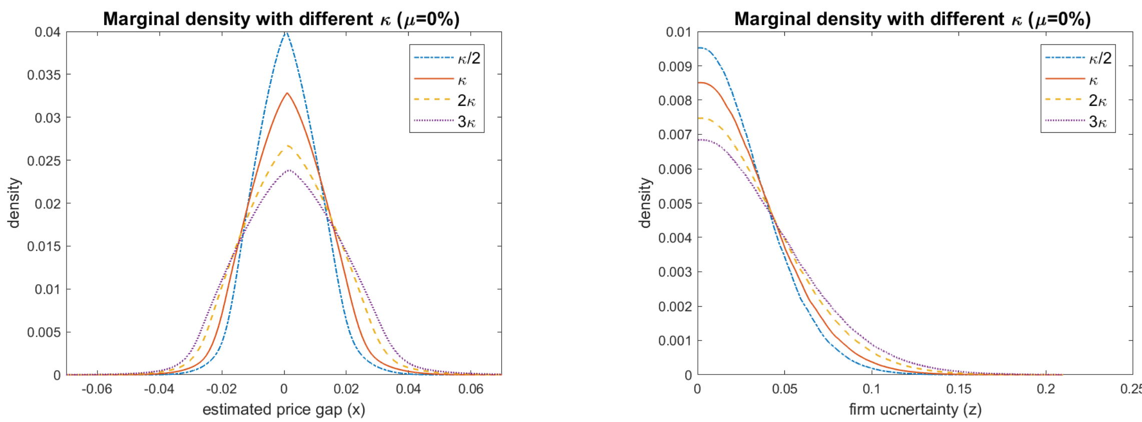
As with χ's experiment, I compute the optimal information acquisition and inaction region with different value of κ, shown in figure 11. The right panel (b) shows that the inaction region expands as the menu cost increases, which is the same as the standard full information menu cost model. With a high menu cost κ, firms are reluctant to adjust the price even if they perceive that their prices are far from the optimal one. Moreover, the value of menu cost affects the firm's information choice. The left panel of figure 11 plots the optimal η for each value of κ. It shows that around the boundary of the inaction region, the firm obtains more information about its price position as κ increases. Intuitively, when menu costs are high, firms want to determine price adjustments based on the most accurate estimates possible.

Furthermore, the change in κ has an impact on the firms' stationary distribution. Figure 12 plots the marginal density of the estimated price gap and its uncertainty with different values of κ. I find that as the menu cost increases, the density around the boundary of the inaction region is monotonically decreasing. It implies that the fraction of firms who pay the information cost to obtain new information and then adjust their prices decreases, which lowers the frequency of price adjustment.



(a) Information acquisition **(b)** Inaction region

Figure 11: Firm's policy functions with different κ



(a) Marginal density of x **(b)** Marginal density of z

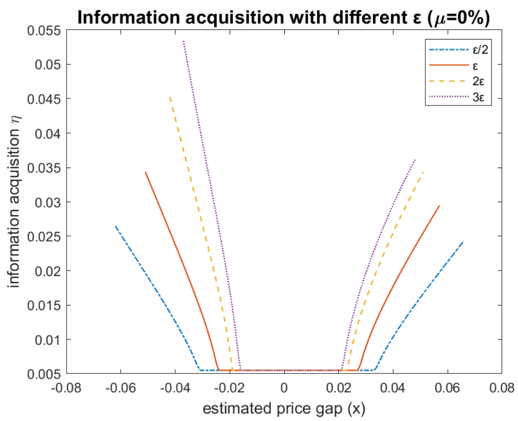
Figure 12: Stationary distribution with different κ

Such an effect of menu cost on the price setting frequency is observed in the standard full information menu cost model. Notice, however, that there is an amplification mechanism generated by the information acquisition choice. As I showed, as the menu cost gets higher, firms who consider that their price gaps are close to the boundary of their inaction regions choose larger η. It means that around the boundary, firms modify their estimates drastically using new information about the idiosyncratic shocks. Thus, the density around the boundary of the inaction region decreases, which amplifies the original effect of menu cost. Since the share of firms who acquire costly information decreases, panel (b) of figure 12 shows that an increase in the menu cost raises the average firm uncertainty .

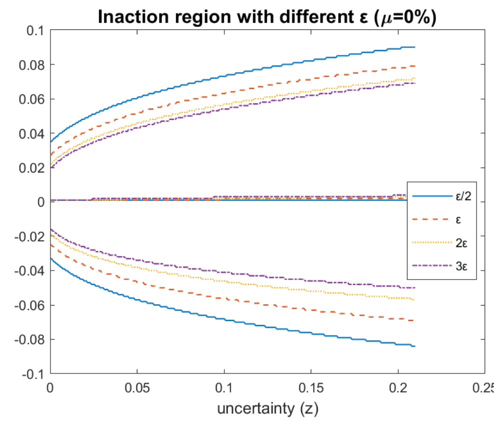
To sum up, as a price adjustment becomes more costly, each firm change its price more carefully and less frequently. Although price adjusting firms acquire much information to conduct accurate estimation, the share of those firms decreases and the entire uncertainty increases.

5.3 Elasticity of Substitution

Finally, I study the effect of a change in the market structure. More precisely, I compute the stationary equilibria with different values of elasticity of substitution ε. Notice that ε represents the elasticity of substitution among goods or the elasticity of demand for each firm's goods.



(a) Information acquisition



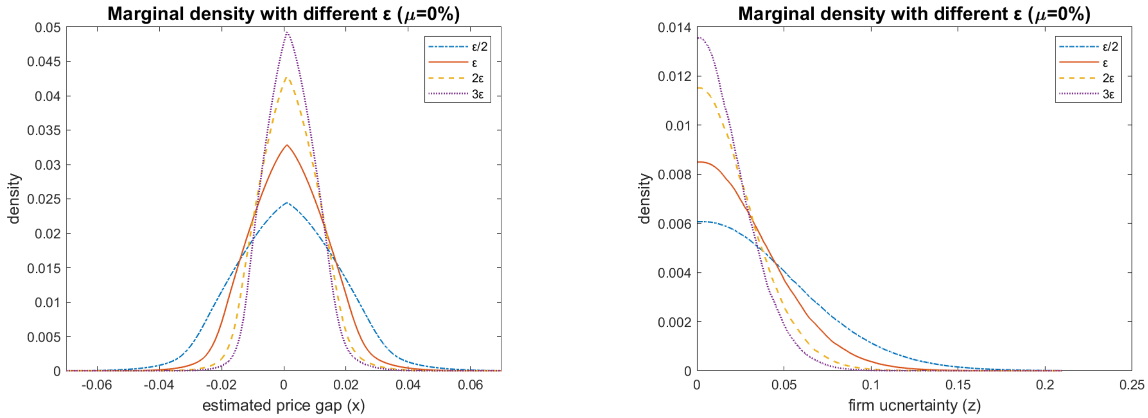
(b) Inaction region

Figure 13: Firm's policy functions with different ε

Figure 13 illustrates the firm's optimal information acquisition and inaction region. Panel (a) shows that as the demand becomes more elastic (large ε), the firm pays more attention to the signals and acquires more information. Moreover, panel (b) indicates that with a higher ε, the inaction region gets narrower. These findings suggest that the firm who faces an elastic demand is willing to pay the information cost for accurate estimates and adjust its price by a small amount.

Since firms become attentive to the costly information, their uncertainty decreases as the elasticity of substitution increases, shown in figure 14. This prediction is consistent with empirical findings, such as Coibion et al. (2018). They find that firms who face more competitors are more likely to be better informed than firms with fewer competitors.

In conclusion, these results of comparative statics indicate that the firm's information acquisition and price setting are strongly linked and the firm responds to changes in the cost of a given decision by changing both behaviors. For instance, a change in the cost of price adjustment affects not only the firm's price setting behavior but also its information acquisition. Furthermore, the market structure also determines each firm's decision and distribution among firms.



(a) Marginal density of x

(b) Marginal density of z

Figure 14: Stationary distribution with different ε

6 Aggregate Propagation

In this section, I analyze the real effect of monetary shock in two theoretical models: the full information benchmark and my rationally inattentive firm model. In both cases, I compute the response of aggregate output and aggregate price to a one-time unexpected permanent increase in the money supply of size $\delta\%$. Suppose that an unexpected shock arrives at time $\tau > 0$, the sequence of money supply is written as

$$\log M(t) = \log M_0 + \mu t \qquad 0 \leq t < \tau$$
$$\log M(t) = \log M_0 + \mu t + \delta \qquad t \geq \tau$$

Since the nominal wage is proportional to the money supply, positive $\delta\%$ shock shifts the stationary distribution of the perceived price gap in the opposite direction. Here, this monetary shock is assumed to be completely observed by all firms because there is no aggregate uncertainty. From the optimal condition of the representative household, I can show that the increase in the money supply is decomposed into the output and price deviations from the stationary equilibrium:

$$\gamma \log \frac{c(t)}{\bar{c}} + \log \frac{P(t)}{\bar{P}(t)} = \delta \quad (\forall t \geq \tau)$$

where $\bar{P}(t)$ is the aggregate price index in the equilibrium which grows at a rate of μ.

Notice that to compute the impulse response function, many previous literature such as Alvarez and Lippi (2014) or Baley and Blanco (2019) assume a simplified environment due to computationally difficulties. In these papers, they assume that a firm's profit function is approximated so that it is independent of the aggregate output, and firms follow the steady state optimal policy along the transition path. Thus, they ignore the general equilibrium feedback effect of firms. In the first part of this section, I follow the previous literature and use this assumption to compute the impulse responses. Then, however, I relax this assumption and consider the case where firms can continuously react and change their policy function during the transition period. To derive the transition dynamics without the above assumptions for simplification, I apply a novel computational method, which is described in Appendix. This part is a methodological contribution of my paper.

In addition to the information structure, I examine the effect of the trend inflation rate and the size of monetary shock δ on the impulse response of aggregate variables. I find that the value of the inflation rate or the shock size does not have much effect when the information is complete, but it has a significant impact on the transition path of output and price under incomplete information.

6.1 Perfect Information Benchmark

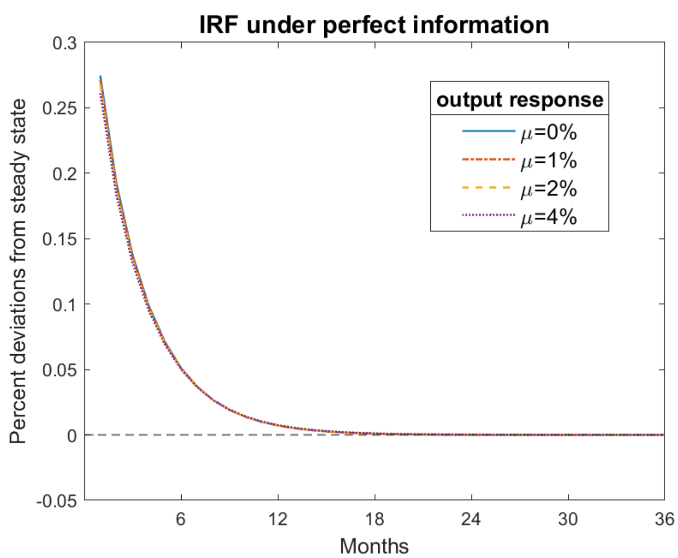


Figure 15: Impulse response of output in perfect information model

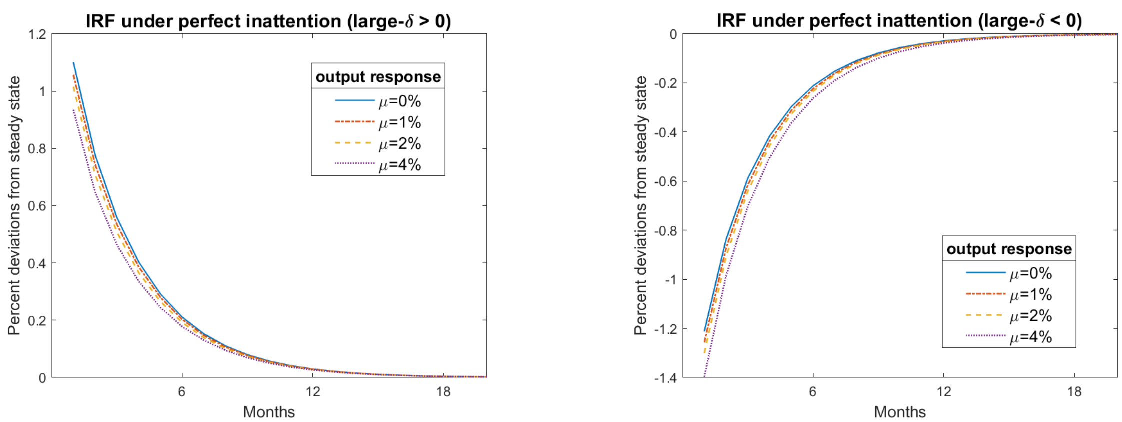
Notes. I plot the output responses to a $\delta = 1\%$ increase in the money supply in the perfect information benchmark model with different level of trend inflation rate.

Firstly, I compute the impulse response function for the benchmark model. Figure 15 plots the impulse responses of the output to the monetary shock of size $\delta = 1$ percent with different levels of trend inflation rate.

The figure shows that in the standard menu cost model, the real effect of monetary shock is small and short-lived: the half-life of impulse response is only three months. This result is due to the large selection effect of price setting behavior, which is well-known mechanism to reduce money nonneutrality in the menu cost model (Golosov and Lucas 2007).

Moreover, I find that the effect of trend inflation is very limited if the shock size is small. This is because the stationary distribution of the price gap is not very responsive to changes in the inflation rate. This finding is in line with Alvarez et al. (2019), they show that when inflation is zero, changes in the inflation rate do not have a first-order effect on the distribution of relative prices.

Next, I consider the effect of the shock size on the output responses. Figure 16 illustrates the output response to the large positive and negative monetary shock ($\delta = 5$ percent). The figure shows that when the shock size is large, increasing the level of trend inflation slightly mitigates the output response to a positive shock and amplifies that to a negative shock, which is consistent with the findings in Alexandrov et al. (2020). Under my calibration, compared to a zero inflation rate, the four percent inflation rate decreases the cumulative impulse response to the positive shock by 16% and increases that to the negative shock by 19%.



(a) Large positive shock

(b) Large negative shock

Figure 16: Impulse response to the large monetary shocks

The mechanism through which trend inflation creates asymmetric effects on the output dynamics is the shape of the stationary distribution of price gaps. The positive inflation rate erodes the firm's price gap and leads to a higher density at the lower boundary of the inaction region. It strengthens the selection effect if the shock is positive. Thus, with a positive inflation rate, a positive shock triggers more firms to adjust prices and the real effect of a positive monetary shock decreases.

Notice that, while there are some differences, the perfect information benchmark generates the small and short-lived real effect of the monetary shock in any value of inflation and in the size of shocks.

6.2 Effect of Information Acquisition

Next, I analyze the output response to the monetary shock with rationally inattentive firms. Figure 17 compares the impulse responses of output in the rational inattentive menu cost model and the perfect information benchmark. The figure shows that the rational inattentive model generates large and persistent real effects of monetary shock: the impact effect is 1.5 times greater than the benchmark model, and the half-life of the output response is about 14 months, 4.6 times longer than the 3 months of the standard menu cost model.

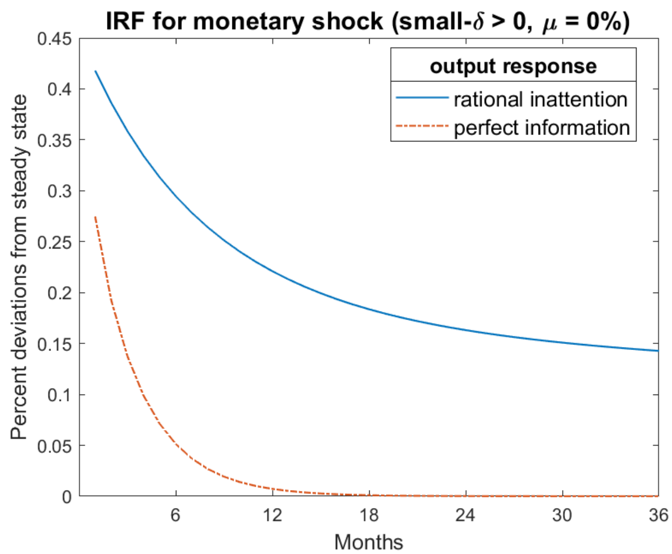


Figure 17: Impulse response of output to 1% monetary shock

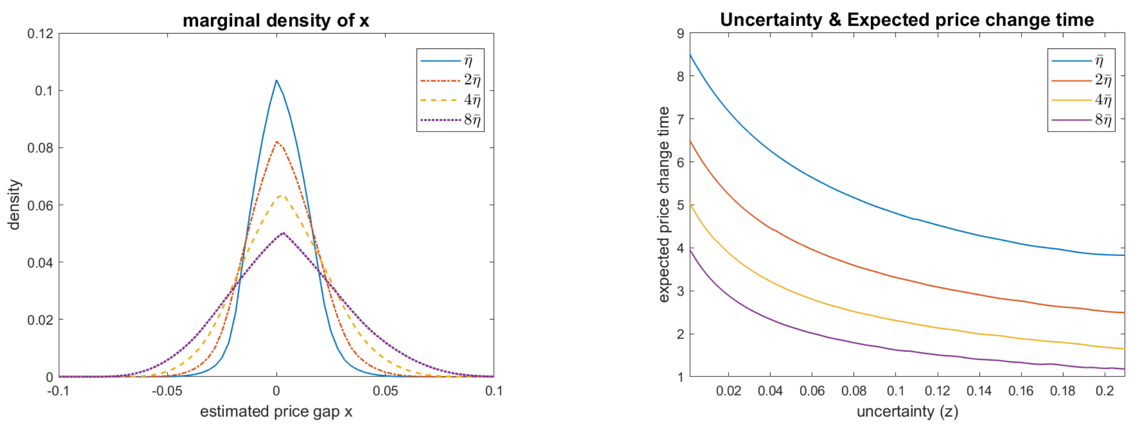
Notes. I plots impulse responses of output to a small ($\delta = 1$ percent) monetary shock under two economies. In both cases, I assume the trend inflation rate is zero and use the calibrated parameter values.

6.2.1 Amplification mechanism

In this subsection, I examine the underlying two mechanisms which generate the large and long-lasting real effect in the rational inattention model: the shape of the stationary distribution and the learning speed of each firm. First, when the inflation rate is zero, the stationary distribution of the perceived price gap becomes leptokurtic. It reduces the selection effect because the density of firms who are close to the boundary of the inaction region decreases. This slows down the rate of adjustment to a shock and makes the real effect larger and more persistent.

Second, due to the firm's information acquisition choice, the expected time for price adjustment depends on the uncertainty and trend inflation rate. I showed that when the inflation rate is zero, it takes firms with any uncertainty a longer time to change prices than when the inflation rate is positive. In addition, when the inflation rate is close to zero, less uncertain firms learn slowly about their idiosyncratic shocks and they take a longer time to adjust prices than more uncertain firms. Thus, the real effect of monetary shock depends on the firm's learning speed: if the firm takes a long time to reach the boundary of the inaction region, the output response gets persistent. This mechanism is new in the literature.

To investigate these mechanisms more precisely, I conduct a counterfactual exercise: compute the impulse response of the output with different values of the lower bound of attention $\bar{\eta}$. This lower bound represents the lowest intensity of attention and affects each firm's learning speed and firms' distribution.



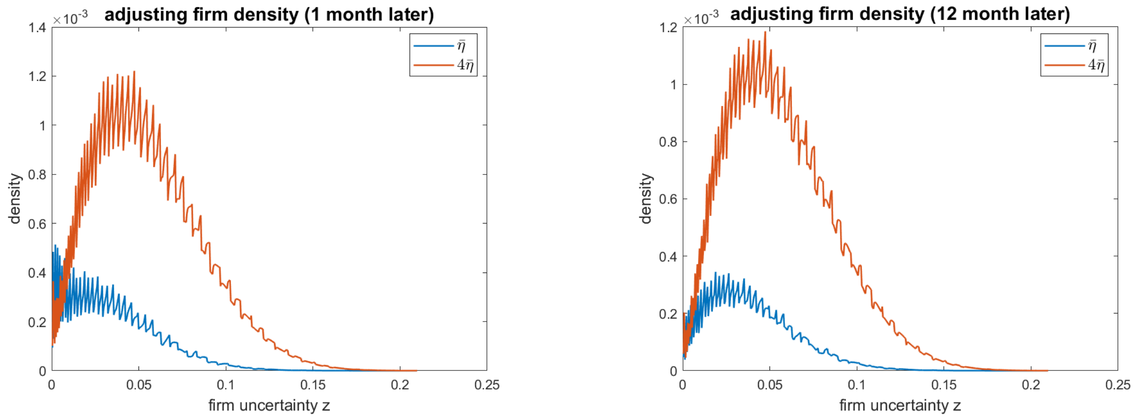
(a) Marginal density of x with different $\bar{\eta}$ **(b)** Expected time for adjustment with different $\bar{\eta}$

Figure 18: Effect of $\bar{\eta}$

Notes. Panel (a) plots the stationary marginal density of estimated price gaps with different values of $\bar{\eta}$. Panel (b) plots the expected time for the next price change, which is derived by the same calculation in section 4.2.1.

I consider counterfactually large values of $\bar{\eta}$. The left panel of figure 18 plots the stationary marginal density of estimated price gap with different $\bar{\eta}$. It shows that the kurtosis of the distribution declines with a large $\bar{\eta}$, which mitigates the first mechanism because the density of firms around boundaries of the inaction region increases. Moreover, firms can receive more information without the information cost, a high $\bar{\eta}$ raise each firm's learning speed. The right panel of figure 18 shows that a high $\bar{\eta}$ shortens the expected time for the next adjustment. Thus, the second mechanism also weakens.

Figure 19 numerically illustrates the change in the two mechanisms according to the value of $\bar{\eta}$. This figure plots the density of adjusting firms who pay the menu cost to adjust



(a) One month after a shock

(b) One year after a shock

Figure 19: Adjusting firm's density

Notes. I plots the density of adjusting firms who get out of their inaction region within a small time interval for each value of z. Panel (a) illustrates the density one month after the shock arrived, and panel (b) does that one year after. In both figures, blue line represents the original $\bar{\eta}$ case and red dotted line is the case of a counterfactual large $\bar{\eta}$.

their prices within a small time interval. The left panel plots the density one month after the shock and the right panel plots that one year after. In both moments, I find that a high $\bar{\eta}$ increases the total number of adjusting firms, and raises the ratio of uncertain firms who learn and adjust prices quickly. For instance, one month later, a high $\bar{\eta}$ raises the density of adjusting firms about four times higher, and the average uncertainty more than six times higher compared to the original calibration.

As a result, the impulse responses of the output to the monetary shock vary according to the lowest intensity of attention. Figure 20 plots the impulse responses to the one percent monetary shock with different values of $\bar{\eta}$. It shows that a high $\bar{\eta}$ generates a relatively small and short-lived real effect, and gets close to the result in the perfect information case. When the lowest intensity is eight times larger than the original value, the impact effect is reduced to 80% of the original result and the half-life is about six months, which is less than half as short as the original one. These results are intuitive because in general, the weaker the information friction is, the closer we get to a perfect information outcome.

To sum up, when firms are rationally inattentive to their idiosyncratic shocks, the aggregate output responds to the monetary shock larger and more persistently. This is because firms endogenously choose their information acquisition, which generates an leptokurtic distribution of estimated price gaps and cross-sectional variation in time for the price adjustment. When the inflation rate is zero, only a small fraction of firms adjust prices, and among those firms there is a concentration on those with low uncertainty. Since low uncer-

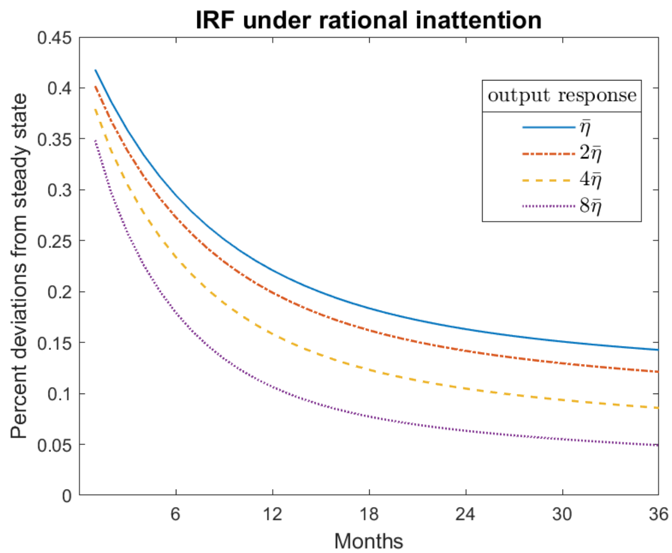
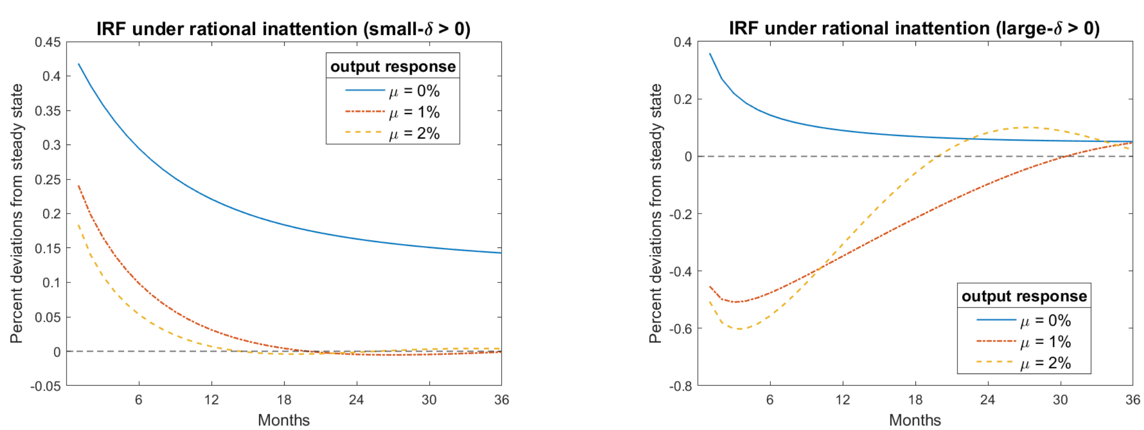


Figure 20: Impulse response with different $\bar{\eta}$

tainty firms have less incentive to update their beliefs, they adjust prices relatively slowly. Combining these mechanisms, the real effect of monetary shock gets larger and lasts longer.

Interestingly, these two mechanisms are affected by the lowest intensity of attention, or equivalently, the amount of information firms can obtain for free. As more information becomes available without paying information costs, the number of firms who adjust prices increases, especially those with high uncertainty, who adjust prices quickly. Consequently, the response of the output to the shock gets weaker. This prediction is a novel part of this paper.

6.2.2 Effect of trend inflation rate and shock size



(a) small monetary shock

(b) large monetary shock

Figure 21: Impulse response of output

Moving on to consider the effect of the trend inflation rate and the size of shock on the impulse responses. Figure 21 plots the impulse response of the output with different inflation rates and the size of shock. The left panel shows that when the shock size is small (δ is one percent), positive inflation rates weaken the output response significantly. The underlying mechanism is the same as the case of $\bar{\eta}$. As I discussed earlier, a positive inflation rate increases the density around boundaries of the inaction region and shortens the expected time of each firm's price adjustment. Thus, the real effect of monetary shock becomes small and short-lived.

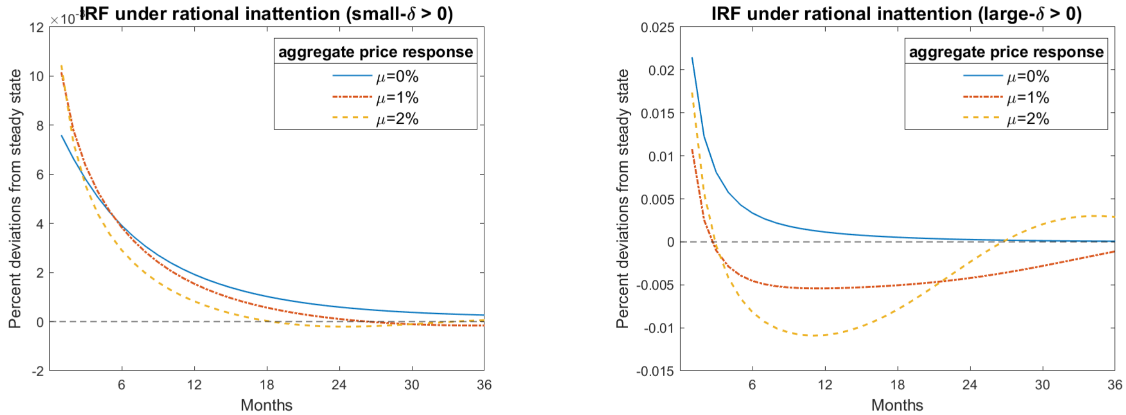
Notice that the causes of these changes in the stationary distribution and price adjustment time are different from the case of $\bar{\eta}$. Here, the reason is that the firm puts more weight on the aggregate trend when it estimates the state when the inflation rate is positive. Because the firm can anticipate that its own price will be smaller than the optimal value due to the positive inflation rate, the distribution of estimated price gaps has a fatter left tail. In addition, a positive inflation rate reduces the time to reach the lower bound of the inaction region, and it shortens the conditional expected time until the next price adjustment.

Furthermore, the right panel of figure 21 shows that when the inflation rate is positive, a large monetary shock (δ is five percent) has conversely negative real effect. This negative output response is explained by two steps. First, when the inflation rate is positive, there is a high density of firms around boundaries of the inaction region, which makes the selection effect become large: when the shock arrives, lots of firms immediately change their prices. Second, adjusting firms choose higher prices than the optimal level due to the positive inflation rate. Thus, when the shock size is sufficiently large, a significant fraction of firms raise prices above the optimal level, causing demand to fall and the aggregate output to drop below its steady-state value.

Notice that this result heavily depends on the assumption that firms can perfectly observe the aggregate variables and thus the size of monetary shock. If I assume that firms also cannot observe the aggregate variables directly, the positive inflation rate and large shock might not generate the negative output response.

I also investigate the aggregate price responses. Figure 22 plots the growth rate of the aggregate price, or the percentage deviation from the equilibrium inflation rate. The left panel shows that when the shock size is small, the positive trend inflation rate slightly strengthens the initial response of the aggregate price and hastens the convergence. This is because the increase in the money supply δ is decomposed into the output and the price deviation from the steady state at any time and a positive inflation rate has a small and short-lived real effect.

On the other hand, the right panel shows that when the shock size is sufficiently large and



(a) small monetary shock

(b) large monetary shock

Figure 22: Impulse response of aggregate price

the inflation rate is positive, the monetary shock generates an oscillatory path of aggregate price. It is caused by a cycle of the density of firms who reach to the boundary of the inaction region: with a positive inflation, the density reaching the boundary is time varying. Because it takes time for a large mass of firms to reach the edge, return to the optimal point, and then go to the edge again, this cycle generates the oscillations of the aggregate price response. Such a mechanism to generate an oscillation of dynamics is called as *echo effect* proposed by Benhabib (e.g. Benhabib and Hobijn 2002).

6.3 Including General Equilibrium Effect

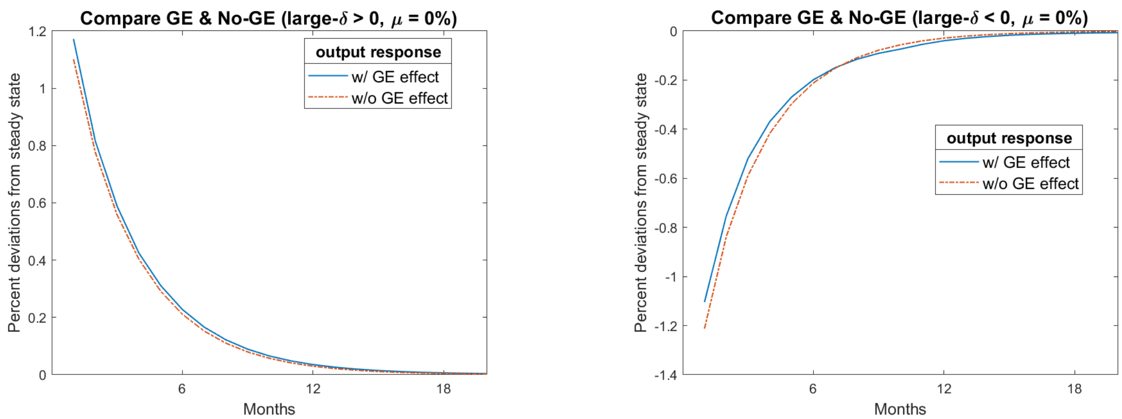
Finally, I investigate the general equilibrium feedback effect and examine the plausibility of assumptions for simplification when calculating the impulse responses.

I compute the same impulse response of the output to the one-time monetary shock in both the perfect information benchmark and imperfect information model. Now, however, I relax the assumption that the firm follows the steady state policy function during the transition. Instead, here I assume that the firm takes the changes in the aggregate variables into account and adjusts its optimal policy. That is, I take the general equilibrium feedback effect fully into consideration.

6.3.1 Perfect information

First, I consider the perfect information menu cost model. Figure 23 compares the output responses to large positive and negative monetary shocks. It shows that there is little difference between the two calculations: for instance, with a positive large shock, the feedback effect increases the impact effect 6% and the cumulative impulse response 5%.

Intuitively, in the perfect information benchmark, the feedback effect only has a small impact on the dynamics of aggregate variables because firms can perfectly expect the future path of the aggregate output and they do not change the price setting decision rules much. Indeed, my numerical calculations show that the firm's optimal policy slightly responds to shocks, but quickly returns to the steady state level.



(a) large positive shock **(b)** large negative shock

Figure 23: Including GE effect, perfect information model

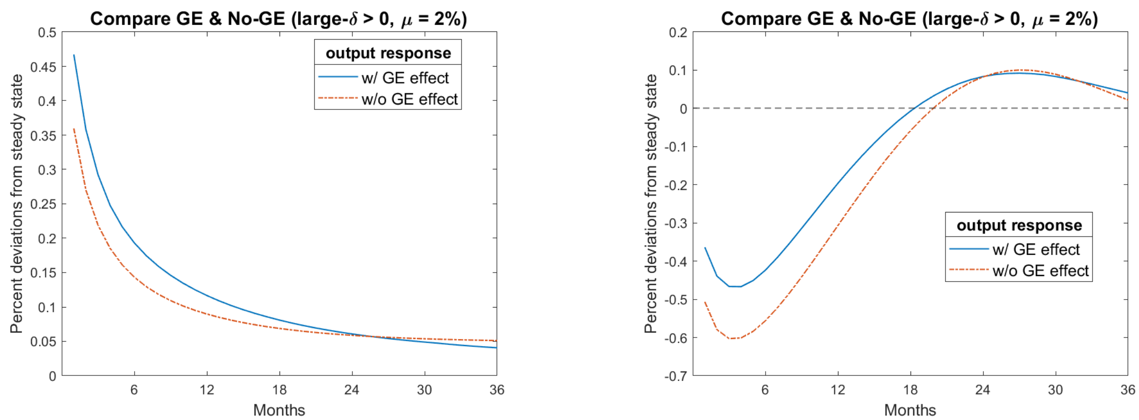
Notes. Blue line represents the impulse response with including the general equilibrium feedback effect, and red dotted line is the impulse response calculated based on simplified assumptions. In both cases, the trend inflation is assumed to be zero.

Thus, the results imply that the general equilibrium effects are negligible for a monetary shock of realistic magnitude in the standard menu cost model. This finding is in line with Alvarez and Lippi (2014).

6.3.2 Imperfect information

Next, I consider the imperfect information model. Figure 24 illustrates a comparison of the output response to large monetary shock under zero and two percent trend inflation rate. In both cases, there is no significant quantitative difference between the impulse responses with and without general equilibrium effect, but qualitatively, the firms' revision of their policy functions has a positive effect on the output response to a positive monetary shock.

The main mechanism underlying this positive effect is a change in the firm's information acquisition choice. After the shock arrives, firms become inattentive to the idiosyncratic signals in two ways: First, the inattentive region in which the firm does not pay the information cost expands. Second, the amount of information acquisition around the boundary of the inaction region declines. Both changes in information choice reduce the learning speed of firms in the inaction region and lower the density of price adjusting firms in the initial



(a) 0% inflation rate

(b) 2% inflation rate

Figure 24: Including GE effect, rational inattention model

period. Because more firms keep the lower price, the aggregate output increases compared to the case without the general equilibrium effect.

Notice that the change in the firm's decision rule is mainly due to the assumption of the monetary shock. I assume that the size of the increase in money supply is perfectly observable for firms with any uncertainty. Thus, firms know that the perceived size of the change in their price gap is not wrong. As a result, firms give less weight to new information from signals before adjusting prices.

7 Conclusion

I investigate the relationship between the firm's price setting and expectation formation, and study the effect of trend inflation rate on these firm's decisions by constructing a new theoretical framework that combines the state-depending price setting problem and the rational inattention problem.

Using the new framework, I find that there is a strong interaction between the firm's price setting and information choice behavior, and the level of trend inflation strongly affects the firm's decision rules and the distribution. Furthermore, I show that the information friction plays an important role in the price setting behavior. It generates the heterogeneity in the size and frequency of price adjustments across firms and a leptokurtic distribution of estimated price gaps. I find that these imperfect information mechanisms amplify the aggregate output responses to monetary shock and thus, generate a large and long lasting real effect of monetary shock.

In addition, I numerically investigate the general equilibrium feedback effect on the impulse responses to monetary shock by using a novel computational method. The results indicate that the general equilibrium effect changes the initial responses of the aggregate variables to some extent, but quantitatively, the simplified assumption provides a good approximation of the impulse responses.

There are several important directions for future research. First, I assume that firms are uncertain only about their idiosyncratic productivity. In contrast, many empirical papers suggest the importance of the aggregate uncertainty. Kumar et al. (2015) find that firms are also uninformed about the aggregate variables, for example, the value of inflation rate or the central bank's policy. Aastveit et al. (2017) find that when the economic uncertainty is high, the effect of monetary policy shock becomes weaker. Thus, further work should be conducted by incorporating the aggregate uncertainty into the firm's information choice behavior, as discussed in Mackowiak and Wiederholt (2009). This extension might have interesting implications for the effect of monetary policy, such as the welfare effect of the information targeting, which can provide the new insight into the validity of two percent inflation rate targeting.

Second, I find some testable micro predictions about the firm's price setting behavior, and also show that the aggregate results are sensitive to the value of the parameters in the firm's information choice. Thus, empirical studies are required to assess my theoretical predictions and estimate the parameter value more rigorously using the firm's price micro data provided, for instance, in Cavallo and Rigobon (2016). These extensions and research remain for future work.

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Computational Appendix

In this appendix, I explain the numerical methods for computing the stationary equilibrium and transition dynamics in the rational inattentive menu cost model. Notice that because the standard menu cost model without information friction can be also solved by following the same procedures, I skip the explanation for the perfect information case. Main references are Azimzadeh (2017), Kaplan et al. (2016) and the numerical appendix of Achdou et al. (2021).

A. Stationary Equilibrium

Consider the case that the functions f, Ξ, Γ do not depend on time. Then, the firm's problem in the stationary equilibrium is given by the following discounted HJBQVI:

$$\max\left\{-\rho V - \mu V_x + \sigma^2 V_z + f(x) + \sup_{\eta \geq \bar{\eta}}\left\{\left(\frac{1}{2}V_{xx} - V_z\right)z\eta - \frac{\chi}{2}(\eta - \bar{\eta})^2\right\}\ ,\ \mathcal{M}V - V\right\} = 0 \quad on\ \mathbb{R}\times\mathbb{R}_+ \tag{19}$$

Note that in this case, the value function V is no longer a function of time and state space but rather a function of state alone. Moreover, as I described in equation (17), the definition of the (discounted) intervention operator $\mathcal{M}$ is given by

$$\mathcal{M}V(x,z) := \sup_{\xi}\left\{V\left(\Gamma(x,\xi), z\right) - \kappa\ ;\ \xi \in \Xi\right\} \tag{20}$$

To solve this HJBQVI, I apply the "penalty scheme" and use the policy iteration method[14].

A.0. Procedures

The computational procedures for the stationary equilibrium are as follows.

1. Guess the value of equilibrium aggregate consumption $\bar{c}$ which determines the coefficient of the instantaneous profit function f.

2. Solve the HJBQVI (19) given the guess and obtain the optimal policy $\{\underline{x}(z), \hat{x}(z), \bar{x}(z)\}$ and $\eta(x,z)$.

3. Derive the stationary density of state (x,z) from the Fokker Planck equation with the firm's policy function.

[14] For details, see chapter 2.3 of Azimzadeh (2017).

4. Calculate the aggregate consumption using the derived density function.
5. Check whether it is sufficiently close to the old one. If not, update the guess and iterate.

I mainly explain the solution methods of the HJBQVI and derivation of the stationary density function.

A.1. Preliminary

Before solving the HJBQVI, I have to approximate the function $V(x,z)$ at $M := N_x \times N_z$ discrete points in the space dimension, $x_i, i = 1, 2, \cdots, N_x$ and $z_j, j = 1, 2, \cdots, N_z$. I denote by Δ_x and Δ_z the distance between grid points [15], and use the short-hand notation $V_{ij} := V(x_i, z_j)$.

We use $\mathcal{D}$ and $\mathcal{D}^2$ to denote the discretization of the first and second partial derivatives so that $V_x(x_i, z_j) \approx (\mathcal{D}_x V)_{ij}$, $V_{xx}(x_i, z_j) \approx (\mathcal{D}_x^2 V)_{ij}$. Precisely, for any matrix $\psi := \{\psi_{ij}\}_{i=1,\cdots,N_x, j=1,\cdots,N_z}$, I define

$$
\begin{aligned}
(\mathcal{D}_x \psi)_{ij} &:= \frac{\psi_{i,j} - \psi_{i-1,j}}{\Delta_x} \\
(\mathcal{D}_z \psi)_{ij}^{+} &:= \frac{\psi_{i,j+1} - \psi_{i,j}}{\Delta_z} \\
(\mathcal{D}_z \psi)_{ij}^{-} &:= \frac{\psi_{i,j} - \psi_{i,j-1}}{\Delta_z} \\
(\mathcal{D}_x^2 \psi)_{ij} &:= \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta_x)^2}
\end{aligned}
$$

where I use the backward difference approximation of the first derivative with respect to x,[16] and let $(\mathcal{D}_z \psi)^{+}$ and $(\mathcal{D}_z \psi)^{+}$ denote the forward and backward difference approximation of the first derivative with respect to z, respectively.

[15] Here, I assume a uniformly spaced grid point. Of course, I can easily apply the nonuniform grid points, and indeed I do when I calculate the transition dynamics in the perfect information benchmark case. To capture the path of firms' optimal triplet more correctly, after deriving the triplet with the uniform grids, I construct the nonuniform grid space in which there are finer grids around the triplet and recalculate using this nonuniform grid space.

[16] This is because I consider the non-negative inflation rate environments ($\mu \geq 0$), and then, the condition of convergence leads to this assumption.

A.2 Intervention Operator

The intervention operator is defined as

$$\mathcal{M}V(x,z) := \sup_{\xi\in\Xi}\{V(\Gamma(x,\xi),z)-\kappa\ ;\ \xi\in\Xi\}$$

Then, we discretized the intervention operator according to

$$(\mathcal{M}V)_{ij} = \sup_{\xi_{ij}\in\Xi^h(x_i,z_j)}\{\text{interp}\,(V,\Gamma(x_i,\zeta_{ij}),z_j)-\kappa\} \tag{21}$$

where we use interp(V,x,z) to denote the value of the numerical solution at (x,z) as approximated by a standard monotone linear interpolant.

To deal with the infinite control space $\Xi=\mathbb{R}$, we take $\Xi^h(x,z)=\{x_1-x,\cdots,x_{N_x}-x\}$ so that

$$\begin{aligned}(\mathcal{M}V)_{ij} &= \max_{\xi_{ij}\in\Xi^h(x_i,z_j)}\{\text{interp}\,(V,x_i+\xi_{ij},z_j)-\kappa\}\\ &= \max_{1\le k\le N_x}\{\text{interp}(V,x_k,z_j)-\kappa\}\\ &= \max_{1\le k\le N_x}\{V_{k,j}-\kappa\}\end{aligned}$$

A.3 Penalty Scheme

The penalty scheme is one of the numerical schemes for HJBQVIs. The basic idea of the penalty scheme is to discretize the "penalized" form of (19). Bensoussan and Lions (1984) proved that, subject to some technical conditions, the solution to the original HJBQVI is the pointwise limit of the solution to the "penalized" form of it. We solve the discretized version of the "penalized" form using the auxiliary control $d_{ij}\in\{0,1\}$ as follows:

$$\sup_{d_{ij}}\left\{-\gamma_{ij}(V,\eta)+f_i+\frac{1}{\epsilon}d_{ij}\left[(\mathcal{M}V)_{ij}-V_{ij}\right]\right\}=0 \tag{22}$$

for $i=1,\cdots,N_x$ and $j=1,\cdots,N_z$. where $\epsilon>0$ is a penalty term and,

$$\begin{aligned}\gamma_{ij}(V,\eta) &:= \rho V_{ij}+\mu(\mathcal{D}_xV)_{ij}-\frac{z_j\eta_{ij}}{2}(\mathcal{D}_x^2V)_{ij}-(\sigma^2-z_j\eta_{ij})(\mathcal{D}_zV)_{ij}\\ \eta_{ij} &:= \max\left\{\frac{z_j}{\chi}\left(\frac{1}{2}(\mathcal{D}_x^2V)_{ij}-(\mathcal{D}_zV)_{ij}^+\right),0\right\}+\bar{\eta}\end{aligned}$$

Here, I use the **Upwind scheme** for the approximation of the first derivative w.r.t. z:

$$(\mathcal{D}_z V)_{ij} = \begin{cases} (\mathcal{D}_z V)^+_{ij}, & if\ \sigma^2 - z_j\eta_{ij} \geq 0 \\ (\mathcal{D}_z V)^-_{ij}, & o.w. \end{cases}$$

and thus,

$$-(\sigma^2 - z_j\eta_{ij})(\mathcal{D}_z V)_{ij} = -|\sigma^2 - z_j\eta_{ij}|\mathbb{1}_{\{\sigma^2 - z_j\eta_{ij} \geq 0\}}(\mathcal{D}_z V)^+_{ij} + |\sigma^2 - z_j\eta_{ij}|\mathbb{1}_{\{\sigma^2 - z_j\eta_{ij} < 0\}}(\mathcal{D}_z V)^-_{ij}$$

where $\mathbb{1}$ is an indicator function. The solution to this problem is obtained by applying so-called policy iteration algorithm.

A.4 Policy Iteration

To apply the policy iteration method, I rewrite the above HJBQVI as a nonlinear matrix equation of the form:

$$\text{find } v \in \mathbb{R}^M \quad s.t. \quad \sup_{P \in \mathcal{P}}\{-A(P)v + y(P)\} = 0 \tag{23}$$

To write the penalty scheme in the form of (23), define the domain of control variables as

$$\mathcal{P}_{ij} := \Xi^h(x_i, z_i) \times \{0, 1\} \times [\bar{\eta}, \infty) \tag{24}$$

and let $P_{ij} := (\xi_{ij}, d_{ij}, \eta_{ij}) \in \mathcal{P}_{ij}$ be a set of control for an arbitrary grid point (x_i, z_j). Thus, in my context, $P = \{\xi_{ij}, d_{ij}, \eta_{ij}\}_{ij}$: $(N_x \times N_z)$ matrix. Then, define the matrix-valued function A and vector-valued function y such that for an arbitrary vector v which is a vectorization of $(N_x \times N_z)$ matrix V: $v = (V_{1,1}, \cdots, , V_{N_x,1}, V_{1,2}, \cdots, V_{N_x,2}, \cdots, V_{N_x,N_z})'$ as follows:

$$[A(P)v]_{ij} = \gamma_{ij}(V, \eta) + \frac{1}{\epsilon}d_{ij}\left[V_{ij} - \max_k V_{kj}\right] \tag{25}$$

$$[y(P)]_{ij} = f_i - \frac{\chi}{2}(\eta_{ij} - \bar{\eta})^2 - \frac{\kappa}{\epsilon}d_{ij} \tag{26}$$

and reshape $y(P)$ to a column vector of length M.

Now, using the finite difference method defined in section A.1., $\gamma_{ij}(V, \eta)$ is written as

$$\gamma_{ij}(V, \eta) = a_{ij}V_{i-1j} + b_{ij}V_{ij} + c_{ij}V_{i+1j} + w^B_{ij}V_{ij-1} + w^F_{ij}V_{ij+1}$$

where

$$
\begin{aligned}
a_{ij} &= -\left(\frac{\mu}{\Delta_x} + \frac{z_j \eta_{ij}}{2(\Delta_x)^2}\right) \\
b_{ij} &= \rho + \frac{\mu}{\Delta_x} + \frac{z_j \eta_{ij}}{(\Delta_x)^2} + \frac{|\sigma^2 - z_j \eta_{ij}|}{\Delta_z} \\
c_{ij} &= -\frac{z_j \eta_{ij}}{2(\Delta_x)^2} \\
w_{ij}^B &= -\frac{|\sigma^2 - z_j \eta_{ij}|}{\Delta_z} \mathbb{1}_{\{\sigma^2 - z_j \eta_{ij} < 0\}} \\
w_{ij}^F &= -\frac{|\sigma^2 - z_j \eta_{ij}|}{\Delta_z} \mathbb{1}_{\{\sigma^2 - z_j \eta_{ij} \geq 0\}}
\end{aligned}
$$

Note that I impose artificial reflecting barriers as follows:

$$
\begin{aligned}
b_{1j} = b_{1j} + a_{1j}, &\quad b_{N_x j} = b_{N_x j} + c_{N_x j}, \\
b_{i1} = b_{i1} + w_{i1}^B, &\quad b_{i N_z} = b_{i N_z} + w_{i N_z}^F, \\
a_{1j} = 0, \quad c_{N_x j} = 0, &\quad w_{i1}^B = 0, \quad w_{i N_z}^F = 0 \quad (\forall i \; \forall j)
\end{aligned}
$$

Then, the "intensity matrix" $\mathbf{A}$ is defined as

$$
\gamma(V, \eta) = \mathbf{A} v
$$

where v is a vector of length $M = N_x \times N_z$ with entries $(V_{1,1}, \cdots, , V_{N_x,1}, V_{1,2}, \cdots, V_{N_x,2}, \cdots, V_{N_x,N_z})'$ and $\mathbf{A}$ is a $M \times M$ matrix which diagonal elements are $(a_{ij}, b_{ij}, c_{ij}, w_{ij}^B, w_{ij}^F)$:

$$
\mathbf{A} = \begin{bmatrix}
b_{11} & c_{11} & 0 & \cdots & 0 & w_{11}^F & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
a_{21} & b_{21} & c_{21} & 0 & \ddots & 0 & w_{21}^F & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & a_{N_x-1\;1} & b_{N_x-1\;1} & c_{N_x-1\;1} & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \ddots & \ddots & a_{N_x 1} & b_{N_x 1} & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
w_{12}^B & 0 & \ddots & \ddots & 0 & b_{21} & c_{21} & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & w_{22}^B & 0 & \ddots & 0 & a_{22} & b_{22} & c_{22} & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & w_{N_x\;N_z-1}^F \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & a_{N_x-1\;N_z} & b_{N_x-1\;N_z} & c_{N_x-1\;N_z} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & w_{N_x\;N_z}^B & 0 & \cdots & 0 & a_{N_x\;N_z} & b_{N_x\;N_z}
\end{bmatrix}
$$

Then, I apply Howard's policy iteration procedure to solve the problem (23):

1. Pick an arbitrary initial guess $v^0 \in \mathbb{R}^M$
2. for $k = 1, 2, \cdots$, iterate the following two steps until v^k converges:

   k-1 Pick P^k such that

$$-A(P^k)v^{k-1} + y(P^k) = \sup_{P \in \mathcal{P}} \{-A(P)v^{k-1} + y(P)\}$$

   k-2 Solve the linear system $A(P^k)v^k = y(P^k)$ to obtain v^k

Azimzadeh (2017) showed that this policy iteration applied to the penalty scheme always converges to the unique solution to (23) under the above construction of $\mathbf{A}$. Hence, by iterating this procedure, I obtain the firm's value function $V(x, z)$ and the associated policy functions $\{\underline{x}(z), \hat{x}(z), \bar{x}(z), \eta(x, z)\}$[17].

A.5 Fokker-Planck equation

After deriving the optimal policy functions from the HJBQVI, I calculate the stationary density function of the state $\phi(x, z)$ by solving the Fokker-Planck (FP) equation and the intervention operator.

Without price adjustment, the FP equation is

$$\frac{\partial \phi}{\partial t} = 0 = (\mu + z\eta_x)\phi_x + (z\eta - \sigma^2)\phi_z + \frac{z\eta}{2}\phi_{xx} + \left(\eta + z\eta_z + \frac{z\eta_{xx}}{2}\right)\phi \tag{27}$$

or its discrete approximation is

$$0 = \tilde{\mathbf{A}}\phi \tag{28}$$

where ϕ is the vector of length M which discretizes the density function and $\tilde{\mathbf{A}}$ is a $M \times M$ matrix that represents the discretized approximation of the FP equation using the finite difference as with the HJBQVI. Here, I apply the upwind method for the first derivative w.r.t. both x and z as follows:

$$(\mu + z\eta_x)\phi_x \approx \begin{cases} (\mu + z\eta_x)(\mathcal{D}_x\phi)^+, & if \;\; \mu + z\eta_x \geq 0 \\ (\mu + z\eta_x)(\mathcal{D}_x\phi)^-, & if \;\; \mu + z\eta_x < 0 \end{cases}$$

[17] The inaction region is given by the set of grid points (x_i, z_j) at which $d_{ij} = 0$. Thus, the thresholds of the inaction region $\{\underline{x}(z), \bar{x}(z)\}$ are approximated by the minimum and maximum values of i such that $d_{ij} = 0$ for each $j = 1, \cdots, N_z$.

and

$$(z\eta - \sigma^2)\phi_z \approx \begin{cases} (z\eta - \sigma^2)(\mathcal{D}_z\phi)^+, & if \;\; z\eta - \sigma^2 \geq 0 \\ (z\eta - \sigma^2)(\mathcal{D}_z\phi)^-, & if \;\; z\eta - \sigma^2 < 0 \end{cases}$$

Thus, discritized FP equation for any ϕ_{ij} is given by

$$0 = a_{ij}\phi_{i-1j} + b_{ij}\phi_{ij} + c_{ij}\phi_{i+1j} + w_{ij}^B\phi_{ij-1} + w_{ij}^F\phi_{ij+1}$$

where

$$\begin{aligned} a_{ij} &= \frac{|\mu + z_j\eta_x|}{\Delta_x}\mathbb{1}_{\{\mu+z\eta_x<0\}} + \frac{z_j\eta_{ij}}{2(\Delta_x)^2} \\ b_{ij} &= \eta + z\eta_z + \frac{z\eta_{xx}}{2} - \frac{|\mu + z_j\eta_x|}{\Delta_x} - \frac{z_j\eta_{ij}}{(\Delta_x)^2} - \frac{|z_j\eta_{ij} - \sigma^2|}{\Delta_z} \\ c_{ij} &= \frac{|\mu + z_j\eta_x|}{\Delta_x}\mathbb{1}_{\{\mu+z\eta_x\geq 0\}} + \frac{z_j\eta_{ij}}{2(\Delta_x)^2} \\ w_{ij}^B &= \frac{|z_j\eta_{ij} - \sigma^2|}{\Delta_z}\mathbb{1}_{\{z_j\eta_{ij}-\sigma^2<0\}} \\ w_{ij}^F &= \frac{|z_j\eta_{ij} - \sigma^2|}{\Delta_z}\mathbb{1}_{\{z_j\eta_{ij}-\sigma^2\geq 0\}} \end{aligned}$$

with the same reflecting barrier assumption. Hence, the diagonal elements of $\tilde{\mathbf{A}}$ are given by $(a_{ij}, b_{ij}, c_{ij}, w_{ij}^B, w_{ij}^F)$.

To introduce adjustment, we now define a binary matrix $\mathbf{M}$ called "intervention matrix" which is the natural discretization of the intervention operator $\mathcal{M}$. $\mathbf{M}$ consists of N_z block diagonal matrices of size $N_x \times N_x$: $\mathbf{M} := diag(\mathbf{M}_1, \cdots, \mathbf{M}_{N_z})$. For each $j = 1, \cdots, N_z$, $\mathbf{M}_j$ is defined as

$$\mathbf{M}_j = (M)_{l,k} = \begin{cases} 1, & if \;\; l \in \mathcal{I}_j, \;\; and \;\; l = k \\ 1, & if \;\; l \notin \mathcal{I}_j, \;\; and \;\; k_j^*(l) = k \\ 0, & o.w. \end{cases} \tag{29}$$

where we let $\mathcal{I}_j$ be the set of grid points in the inaction region for the block diagonal grid points $l, k = (1, \cdots, N_x) \times j$, and denote by $k_j^*(l)$ the optimal return point with z_j that is reached from point $l = (1, \cdots, N_x) \times j$ upon adjustment.

To find the stationary distribution for (x, z), I use the "splitting algorithm" based on Kaplan et al. (2016). I split the step of finding ϕ^{n+1} given ϕ^n into two steps:

1. Given ϕ^n find $\phi^{n+\frac{1}{2}}$ from

$$\phi^{n+\frac{1}{2}} = \mathbf{M}'\phi^n \tag{30}$$

2. Given $\phi^{n+\frac{1}{2}}$ find ϕ^{n+1} from

$$\begin{aligned} \frac{\phi^{n+1} - \phi^{n+\frac{1}{2}}}{\Delta_t} &= \mathbf{M}'\tilde{\mathbf{A}}\phi^{n+1}, \\ \Leftrightarrow \left[\mathbf{I} - \Delta_t \mathbf{M}'\tilde{\mathbf{A}}\right]\phi^{n+1} &= \phi^{n+\frac{1}{2}} \end{aligned} \tag{31}$$

Iterate until ϕ^n converges to the stationary distribution ϕ^*, where Δ_t is an iteration step size that determines the convergence speed.

A.6 Update the guess of c

Finally, compute the new aggregate consumption c^* using obtained stationary marginal distribution of x: $\phi^*(x)$

$$c^* = \left(\frac{\varepsilon - 1}{\alpha\varepsilon}\right)^{\frac{1}{\gamma}} \left[\int e^{(1-\varepsilon)x}\phi^*(x)dx\right]^{\frac{1}{\gamma(\varepsilon-1)}} \tag{32}$$

Then, calculate again the HJBQVI and FP equation given the new consumption c^* until the value of the aggregate consumption converges.

B. Transition Dynamics

Next, I explain the computation procedures for the transition dynamics including the general equilibrium feedback effects. The sequence of the value function along the transition $t \in [0, T]$ is given by the following discounted HJBQVI:

$$\max\left\{V_t - \rho V - \mu V_x + \sigma^2 V_z + f(t,x) + \sup_{\eta \geq \bar{\eta}}\left\{\left(\frac{1}{2}V_{xx} - V_z\right)z\eta - \frac{\chi}{2}(\eta - \bar{\eta})^2\right\}\ ,\ \mathcal{M}V - V\right\} = 0 \ \ on\ [0,T) \times \mathbb{R} \times \mathbb{R}_+ \tag{33}$$

$$V(T,\cdot,\cdot) = V^* on\ \mathbb{R} \times \mathbb{R}_+ \tag{34}$$

Now, the functions f and Γ depend on time and the second equation is the terminal condition (V^* is the stationary equilibrium value function).

The detailed calculation method is the same as the one described in the previous chapter, and thus I only explain the procedure here.

B.0 Procedures

The computational procedures for the transition dynamics are as follows.

1. Guess the transition paths of aggregate consumption $c(t)$.
2. Given guessed paths, derive the sequence of value functions $\{V^t\}_{t=1}^{T}$ by solving the HJBQVI backward with the terminal condition: at each time, derive V^t given the value of V^{t+1}.
3. Derive the sequence of density function in each time step by using the Fokker-Planck equation and the sequence of the policy functions $\{\underline{x}(t,z), \hat{x}(t,z), \bar{x}(t,z), \eta(t,x,z)\}$, starting from the initial distribution which is the density immediately after the shock has arrived, but before firms responded to it: derive phi^{t+1} given ϕ^t.
4. Calculate the path of aggregate consumption $c(t)$ using derived density functions.
5. Check whether these paths are adequately close to the old ones. If not, update the guess and iterate.

The difference from the calculation of equilibrium is that I guess the path of the aggregate consumption rather than the steady state value. At each time step, the HJBQVI can be solved by the same numerical method as the stationary equilibrium case. The sequence of density functions is derived forward from the initial distribution by applying the splitting algorithm one at a time.