Model-Based Estimation of Sovereign Default

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Abstract

We estimate a canonical sovereign default model from Arellano (2008) for Argentina via maximum simulated likelihood estimation to understand how well it performs in terms of predicting default events. The estimated model accounts for the overall default patterns of Argentina and closely matches the default data. Out-of-sample forecasting shows that the model performs better than a logit model in predicting the onset of default events. In terms of the business cycle statistics, the findings of the model are consistent with the data and Arellano (2008), with some caveats.

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1 Introduction

How informative are default risks estimated from a stochastic general equilibrium sovereign default model? There is an extensive theoretical literature on sovereign debt crises that builds on the endogenous sovereign default model of Eaton and Gersovitz (1981). Aguiar and Gopinath (2006) and Arellano (2008) are the first studies to extend the Eaton-Gersovitz framework to quantitatively account for the patterns regarding default and business cycles in emerging market economies. While there is a large literature that quantitatively analyzes different aspects of default based on the endogenous sovereign default model, our paper is the first attempt to estimate this model. In this paper, we formally estimate the Arellano (2008) model for Argentina to analyze how well the estimated model explains the actual default events and what parameter values fit the data.

To estimate the model, we apply a structural estimation method for discrete choice dynamic programming models (e.g., Keane and Wolpin, 2009, Train, 2009, and Keane, Todd, and Wolpin, 2011). This method is commonly used in empirical industrial organization and labor literature. To the best of our knowledge, this paper is the first to apply this method to a sovereign default model with an endogenous default decision in the international finance literature.[1] The baseline model has a discrete default choice with two types of uncertainty: uncertainty with respect to the debtor country's output and uncertainty with respect to the timing of regaining market access once the debtor country defaults. Our estimation uses only output and default data, allowing measurement (forecast) error in the observed default variable.

The estimation of the model gives us the set of parameter estimates that has the best fit to the default and output data in Argentina. We find that the estimated value of risk aversion is quite high and the discount factor is on the lower end compared to the values used in the literature. We then estimate the model by fixing the risk aversion parameter and the discount

[1] Bi and Traum (2012) estimate a sovereign default model, where default is not modeled as an endogenous decision but as a probabilistic event based on whether debt exceeds a stochastic fiscal limit.

factor at standard values used in the literature to see how the model predictions change depending on the parameter values.

We find that the model-implied default decisions, with either set of estimates, account for the overall default patterns in Argentina, especially the onset of default events in 1982 and 2001. The model-implied default probability continues to stay high during the periods in which Argentina was excluded from financial markets following the default events and decreases as the country regained access to markets, matching the default data closely.

When compared to the logit-based models that have been commonly used in the early warning systems literature, the estimated sovereign default model performs better in predicting default occurrence. When the model-implied default probability is used as a regressor in a logit estimation, it has a high explanatory power for observed default events. Additionally, pseudo out-of-sample forecasting shows that the baseline model predicts the onset of the 2001 default much better than a comparable logit model, when default forecasts are computed conditional on the current year's output. On the other hand, neither model predicts the onset of the 2001 default when forecasts are computed conditional on previous year's output. In other words, reliably updating output information prior to default is a key to forecast default events.

We also find that the model-implied business cycle statistics are consistent with the data and Arellano's (2008) findings, provided that we impose a parameter restriction on the risk aversion coefficient and the discount factor to equal their commonly calibrated values in the literature. Without such a restriction, the model fails to account for the higher volatility of consumption relative to output as well as the countercyclicality of the trade balance, and it gives a very high volatility for the spread. Hence, the set of estimates that best fit the data in terms of default and output patterns cannot fully account for the business cycle properties observed in the data. We believe that incorporating theoretical developments from the sovereign default literature, such as long-term bonds and debt renegotiation, is necessary to improve the model's accountability for business cycle statistics, which we intend to address in future work.

There is a broad theoretical literature on sovereign default that extends Arellano (2008), and analyzes various aspects of sovereign default. Aguiar and Gopinath (2006) analyze the effects of using a productivity process characterized by a stochastic trend, Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) introduce long-term bonds to a sovereign default model, Yue (2010) incorporates debt renegotiation and endogenous debt recovery into a sovereign default model, Mendoza and Yue (2012) propose a model that endogenizes the output declines observed in sovereign default episodes and Na, Schmitt-Grohe, Uribe, and Yue (2017) introduce downward nominal wage rigidity to explain the empirical regularity that sovereign defaults are accompanied by large devaluations of the nominal exchange rate. Our paper contributes to this literature by structurally estimating the basic endogenous sovereign default model to understand how well it performs in terms of predicting default events. The estimation results also enable us to compare the parameter estimates that fit the data with the values used in the existing literature.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 explains the data and estimation strategy, and Section 4 presents the estimated results. Section 5 provides further discussion on parameter estimates and the model-implied business cycle properties. Section 6 concludes.

2 Model

This section explains the key features of Arellano (2008), which we use as the baseline model in our estimation. We first discuss the sequence of decisions in the model. We then explain how default decision and default risk are modeled. The details of this model are provided in Appendix A.

2.1 Sequence of Decisions

In period t, a country faces debt obligation $-B_t$. It then observes output, y_t, the log of which follows an AR(1) process. If the country had repaid in the previous period, then it would be able to choose to repay or default in period t, denoted as $d_t = 0$ and $d_t = 1$ respectively. If the country chooses to repay, then it also decides how much it borrows in that period ($-q(B_{t+1}, y_t)B_{t+1}$), where q is the price of asset B. If it chooses to default, it can write off its debt obligations at the expense of losing a fraction of output and being excluded from world financial markets for a stochastic number of periods. The state variables (d, B, y) are thus sequentially determined by

$$(d_{t-1}, B_t) \to y_t \to (d_t, B_{t+1}).$$

2.2 Default Probability and Default Decision

The country decides whether to repay its debt or default by comparing the value function under default (V^D) with the value function under repayment (V^R). Thus, the default decision of the country is given by

$$d_t = \begin{cases} 1, & \text{if } V^D(y_t) > V^R(B_t, y_t) \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where B_t is pinned down by the savings policy function of $B(B_{t-1,}y_{t-1})$.

The country's choice of B_t in period $t-1$ implies a default probability for period t conditional on $(d_{t-1}, B_{t-1}, y_{t-1})$, i.e., before y_t is observed. The default probability is given by

$$\begin{aligned} \Pr(d_t = 1 | d_{t-1} = 0, B_{t-1}, y_{t-1}) &= \delta(B(B_{t-1}, y_{t-1}), y_{t-1}), \\ \Pr(d_t = 0 | d_{t-1} = 0, B_{t-1}, y_{t-1}) &= 1 - \delta(B(B_{t-1}, y_{t-1}), y_{t-1}), \\ \Pr(d_t = 1 | d_{t-1} = 1, B_{t-1}, y_{t-1}) &= 1 - \lambda, \\ \Pr(d_t = 0 | d_{t-1} = 1, B_{t-1}, y_{t-1}) &= \lambda, \end{aligned} \tag{2}$$

where λ is the exogenous probability of regaining access to financial markets for a country that has previously defaulted, and δ is defined by

$$\delta(B_t, y_{t-1}) = \Pr\big(y_t \in I(B_t)\big).$$

$I(B_t)$ is the set of y's for which default is optimal for B_t, defined as

$$I(B_t) = \{y_t \in \mathcal{Y}: V^D(y_t) > V^R(B_t, y_t)\}.$$

The default decision is made after the current output, y_t, is realized. Thus, the model-implied default decision for d_t is a nonlinear function of (d_{t-1}, B_t, y_t),

$$d_t = d(d_{t-1}, B_t, y_t).$$

If the country had defaulted in period $t-1$, it would not be able to borrow in period t. The country can regain market access with a fixed probability λ in period t.

3 Data and Estimation Strategy

3.1 Data

We use annual data for Argentine output and default periods for our estimation. For output (y), we use real GDP at constant national prices for Argentina from Penn World Table 9.0 (Feenstra, Inklaar and Timmer, 2015).[2] We remove a stochastic trend from the log of the real GDP series for 1950-2014 by applying the Hodrick-Prescott (HP) filter (with the smoothing parameter equal to 100) and then use the detrended component as $\ln(y)$.

For the default variable (d), we construct a dummy variable that takes the value 1 under default years and zero otherwise following the default years identified by Reinhart (2010) and Beers and Chambers (2006). Specifically, we set the default periods as 1951, 1956-1965, 1982-1993, and 2001-2005.[3]

The black solid line in Figure 1 plots output with default years in the shaded areas. Table 1 provides the summary statistics. We set the sample period as 1952-2010, which starts

[2] We have obtained it through FRED, and its series ID is RGDPNAARA666NRUG. The series is available from 1950.

[3] Reinhart (2010) covers the period 1819 to 2009 for Argentina. Beers and Chambers (2006) cover a more recent period starting in 1980 and identify the same default years as Reinhart (2010) for this period. Uribe and Schmitt-Grohé (2017) provide a summary table (Table 13.19) on sovereign default dates. Since the output series we use starts in 1950, we use the defaults identified in the period after 1950.

after the first observed default of 1951 and excludes the last several years after HP filtering the data to address the end of sample problem.

[Figure 1]

[Table 1]

We do not use debt data for our estimation because we can compute a model-implied debt path given the paths of default and output from the data.[4] Further, publicly available aggregate debt stock data may lack accuracy if they fail to account for all publicly guaranteed debt outstanding, debt reductions, and reschedules.

3.2 Estimation Strategy

We introduce i.i.d. measurement errors into the regime variable to allow the model-implied default path to deviate from the observed default events. Specifically, we assume that $\Pr(d_t^o = 0 \mid d_t = i) = a_i$ for $i = 1\ or\ 0$ where d_t^o denotes the observed default behavior in the data with $d_t^o = 1$ corresponding to default and $d_t^o = 0$ corresponding to repayment in year t. The superscript "o" indicates that the corresponding variable is *observed* in the data. The state space representation is non-Gaussian and nonlinear as follows.

$$
\begin{aligned}
&\ln(y_t^o) = \ln(y_t), \\
&d_t^o = \begin{cases} 0 & if\ d_t = i\ \&\ u_t < a_i, \quad for\ i = 0, 1 \\ 1 & otherwise \end{cases}, \\
&\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t, \\
&d_t = d(d_{t-1}, B_t, y_t), \\
&B_t = \begin{cases} B(B_{t-1}, y_{t-1}) & if\ d_t = 0 \\ 0 & o/w \end{cases}, \\
&u_t \sim i.i.d.\ uniform\ (0,1), \\
&\varepsilon_t \sim i.i.d. N(0, \eta),
\end{aligned} \tag{3}
$$

[4] Specifically, the model implied debt path can be computed using the policy function and the lagged state variables as $B_t = B(B_{t-1}, y_{t-1}) \times 1_{\{d_{t-1}=0\}}$.

where the first two equations are observation equations and the remaining three equations are the state equations. The functions f and g are highly nonlinear.

We apply a maximum simulated likelihood method to estimate the model.[5] Let $D^o \equiv \{d_t^o\}, D \equiv \{d_t\}$ and $Y \equiv \{y_t\}$. The joint distribution of D^o and Y implied by the model can be written as

$$\begin{aligned} P(D^o, Y; \boldsymbol{\theta}) &= P(D^o|Y)P(Y), \\ &= \left[\int P(D^o, D|Y)dD\right]P(Y), \\ &= \left[\int P(D^o|D, Y)P(D|Y)dD\right]P(Y), \\ &= \left[\int P(D^o|D)P(D|Y)dD\right]P(Y), \\ &\approx \left[\sum_i P(D^o|D_i)P(D_i|Y)\right]P(Y), \end{aligned} \tag{4}$$

where $\boldsymbol{\theta}$ is the set of model parameters: σ (risk aversion), r (risk-free rate), β (discount factor), λ (reentry probability), ρ and η (coefficients in the output equation), $\overline{y}$ (output cost), and B_0 (initial asset level). The log likelihood function is

$$\ln P(D^o, Y; \boldsymbol{\theta}) = \ln \sum_i [P(D^o|D_i)P(D_i|Y)] + \ln\ P(Y). \tag{5}$$

The difficulty is that there is no analytical representation of $P(D_i|Y)$. However, we can simulate D_i and compute $P(D_i|Y)$ from the model.[6] Thanks to the parsimonious model feature that there are only eight parameters and many of them have specific ranges, we can carry out simulations for all possible parameter-value combinations with reasonably fine and widely ranged grids.

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[5] See Keane and Wolpin (2009), Train (2009), and Keane, Todd, and Wolpin (2011) for details.

[6] Specifically, the steps of calculation are as follows. Step 1: Given data Y and the model, simulate D_i from distribution $P(D_i|Y)$ many times; Step 2: Given data D^o, calculate forecast error probability $P(D^o|D_i)$ for each simulation D_i; Step 3: Sum up $P(D^o|D_i)P(D_i|Y)$ over simulations. We further explain our numerical method in Appendix D.

In the estimation, for simplicity, we assume no measurement errors in the initial year ($d_{1952} = d^o_{1952}$). Further, we fix $r = 0.025$ (the historical average of real interest rate data series[7]) to make the estimated results comparable to observed levels of r. Lastly, the initial debt level -B_0 is set to zero assuming that there was no external borrowing during the 1951 default.

4 Estimated Results

4.1 Parameter Estimates

We find the unique parameter set that achieves the highest likelihood function value in our numerical maximization framework (see Appendix D for details). The estimated parameters are shown in the first column in Table 2 together with the asymptotic standard errors of the estimated parameters computed using the score vector for observations (see Proposition 7.9 in Hayashi (2000) for details). For comparison, we report the annualized calibrated parameter values used by Arellano (2008) in the last column.

[Table 2]

We find that the estimated risk aversion is quite high ($\sigma = 8$) compared to the value commonly used in the literature ($\sigma = 2$). As a result of this high σ, the average of simulated consumption volatility turns out to be lower than that of simulated output volatility (see Section 4.4 for a discussion of the business cycle properties). Second, the estimated discount factor is quite low ($\beta = 0.58$).[8] While the low β is consistent with the values used in some

[7] We measure the risk-free interest rate as the nominal interest rate (three-year US Treasury securities) minus the average inflation rate (using the GDP deflator) over the current and subsequent two years in the US. This series is available from 1954.

[8] We still obtain a low β value even when we exclude the coup d'etat era of the 1950s and 60s from the sample period.

recent studies (Mendoza and Yue, 2012 and Na et al., 2017), it is lower than the value implied by the Euler equation[9] at the steady state,

$$\beta = \left[(1+r)\left(\frac{\delta^*}{1-\delta^*}\left(\frac{c^{R*}}{c^{D*}}\right)^{\sigma} + 1\right)\right]^{-1}, \tag{6}$$

where superscript $*$ indicates the steady state values. Suppose $c^{R*}/c^{D*} = 1.03$ (roughly in line with Arellano's (2008) calibrated value of $\overline{y}$), $r = 0.025$ (the historical average of the real interest rate data series mentioned above), $\sigma = 2$ (the commonly used calibrated value), $\delta^* = 0.12$ (an approximate default probability computed as the number of defaults divided by the number of years under the repayment state for our sample period). Then, the implied β is 0.85, notably higher than the estimated value of 0.58.

Another value that is quite different from the calibrated value used by Arellano (2008) is λ (probability of reentry). The average duration of observed default years is 9 years[10] in our sample, and the estimated value of λ (0.12) gives a 9-year default duration on average. Arellano (2008) sets this value to a much higher value, 0.73, to match the volatility of the trade balance.

The value of $\overline{y}$ (output cost, 0.99) is consistent with the calibrated values in the literature. The estimated coefficients for the output dynamics (ρ and η) are consistent with the simple AR(1) estimates. The relatively low value of ρ (0.55) reflects the low persistency of

[9] The Euler equation is given by,

$$1 = \frac{\beta}{q_t} E_t\left[d_{t+1}\frac{u'(c_{t+1}^D)}{u'(c_t^R)} + (1-d_{t+1})\frac{u'(c_{t+1}^R)}{u'(c_t^R)}\right] = \frac{\beta(1+r)}{1-\delta_t} E_t\left[d_{t+1}\left(\frac{c_t^R}{c_{t+1}^D}\right)^{\sigma} + (1-d_{t+1})\left(\frac{c_t^R}{c_{t+1}^R}\right)^{\sigma}\right]$$

where the second equality holds by the CRRA utility and bond pricing equation with risk neutral lenders. If c^R and c^D are constant at the steady state, the above equation reduces to $\beta = \left[(1+r)\left(\frac{\delta^*}{1-\delta^*}\left(\frac{c^{R*}}{c^{D*}}\right)^{\sigma} + 1\right)\right]^{-1}$, where the superscript * indicates the steady state values.

[10] Uribe and Schmitt-Grohé (2017) review the existing estimates of years of exclusion from credit markets after default and find that, on average, countries regain full access to credit markets 8.4 years after emerging from default.

output at annual frequency obtained via HP filtering. The estimate of a_1 (the probability that the repayment is observed in the data, given that the model implies default) is 0. Thus, we observe defaults whenever the country chooses to default in the model. The estimate of a_0 (the probability that the observed default variable is repayment given that the model-implied default variable also indicates repayment) is 0.90. This value being lower than 1 reflects that the model fails to predict the 1956 default. These two measurement error-related parameters are not needed in our simulations, as discussed below.

Given that the estimates of σ and β are quite far from the values commonly used in the literature, we also estimate the model by fixing $\sigma = 2$ and $\beta = 0.8$ (the calibrated values used by Aguiar and Gopinath, 2006).[11] We call this the restricted estimation and report the estimated parameters in the second column of Table 2. The parameter value that is mainly affected by this restriction is the value of λ (probability of reentry). In the restricted estimation, λ equals 0.49, which is higher than the value of λ that we get in the unrestricted estimation but still lower than the value used by Arellano (2008).

Why do the unrestricted estimates give such a high σ and a low β? The unrestricted estimate for λ (probability of reentry) is consistent with the observed duration of default years but is much lower than the restricted estimate. The lower λ implies a higher penalty upon default; as a result, the country has less incentive to default. To offset this diminished incentive, the model parameters adjust and give a combination of high σ and low β. The higher the σ or the lower the β is, the greater is the incentive for the country to default.

4.2 Default Probability Function

Figure 2 shows the estimated default probability function, $\delta(B_t, y_{t-1})$, computed with the restricted estimates. The figure plots the default probability as a function of lagged output, y_{t-1}, for three different levels of B_{t-1}. For a given B_{t-1}, the figure shows that the relationship

[11] A reasonable value of σ for a small open economy might be higher than 2 (Reinhart and Végh, 1995). When we re-estimate the model fixing σ=5, the implied default probabilities turn out to be very similar to those of the unrestricted estimates, and thus, they are not reported here.

between the default probability and lagged output is non-monotonic. The default probability first decreases with increasing output, but then it jumps up as the country chooses to borrow more, i.e., -B_t increases, which leads to a jump in the probability of default. The spikes in the default probability correspond to output levels at which the country increases its debt level. The figure also shows that the default probability increases with a higher level of the initial debt (-B_{t-1}), holding output constant, since the new debt level increases with the starting value of debt.

[Figure 2]

4.3 Default Probabilities

We call the probability of the default outcome after y_t (the current output) is realized the ex-post default probability and the probability of default conditional on y_{t-1} the ex-ante default probability. Figure 3 plots the ex-post default probabilities, which are computed as averages of 10,000 simulations, for the unrestricted and restricted parameter estimates. The ex-post default probability equals either 0 or 1 for a country that has access to financial markets. Since the default decision is made after y_t is observed, the default outcome becomes a certain event given the default decision. Thus, for a country that has access to financial markets, the probability of default equals 1 if the country chooses to default and zero if repayment is chosen. For a country in autarky, on the other hand, the probability of remaining in the default state or not depends on the exogenous probability of regaining access to markets, λ. Therefore, the simulated ex-post default probability fluctuates between zero and one after the decision to default due to the exogenous probability of reentry, as seen in the figure.

[Figure 3]

With either set of estimates, the timing of the default decisions is quite similar as seen by the comovement of the two lines. The model matches the observed default events in 1982 and 2001 with the ex-post default probability equaling 1 in these years. The default probability continues to remain high in the years following the default events, identified as default years in the data, even though it fluctuates due to the exogenous reentry probability. The unrestricted estimation gives a higher default probability throughout the default years since the value of λ (probability of reentry) is much lower in the unrestricted estimation than in the restricted estimation (0.12 versus 0.49). With a low λ value, the probability of staying in autarky after the decision to default remains higher, as shown by the smaller swings in the blue line. The low λ value also leads to a slower decline in the probability of default once it increases. As a result, the default probability predicted by the unrestricted estimates in the years of repayment (the white areas) is higher than the restricted estimates. With the restricted estimates, the ex-post default probability falls close to zero in 1994 and 2006, when Argentina regained access to financial markets, matching the data very closely. During the repayment years, it does not fluctuate with changes in output and stays close to zero since the country remains below the model-implied endogenous debt ceiling. Hence, the restricted estimation matches the default status of the country better than the unrestricted estimation during the repayment years, as shown by the lower values of the default probability in the repayment years. The unrestricted estimates, on the other hand, fit the data better during the default years, with the probability of default remaining high during these periods.

In the case of the default in 1956, the model does not predict a default since output does not decline before or during the default year (see Figure 1). Instead, the model predicts a default with some lag in 1958, when we see a decline in output. The default in 1956 follows a political crisis and a coup d'état, after which the new government reached an agreement with the lending nations to reschedule its debt. In this sense, the 1956 default seems to be different from the other defaults, and it does not coincide with an output decline.

Figure 4 compares the ex-post and ex-ante default probabilities for the restricted set of parameter estimates. The red solid line shows the ex-post default probability, and the blue dashed line shows the ex-ante default probability. The ex-ante default probability, which is the probability of default conditional on y_{t-1}, moves similarly to the ex-post default probability in the default years and reaches its highest values during these periods, which is consistent with the data. However, it usually follows the ex-post default probability with a lag. This lagged pattern suggests that the decline in output observed at the time of default events is important for the model to predict a default. It is also the case that the ex-ante default probability increases during some years in the repayment periods, showing a heightened risk of default based on the previous period's output.

[Figure 4]

4.4 Business Cycle Properties

Table 3 compares the moments related to consumption, interest rate spread and net exports with those from the data and Arellano (2008). For each of these variables, we compute the average of 10,000 simulated paths given the output data series and the restricted and unrestricted parameter estimates. We report the moments for both the whole period and the repayment periods.

[Table 3]

The statistics computed with the restricted estimates ($\sigma = 2$ and $\beta = 0.8$) are broadly consistent with the data and Arellano (2008). Specifically, consumption is more volatile than output, the volatility of the spread is very close to the data in the repayment subsample and the trade balance is countercyclical, which is again stronger in the repayment subsample. Figure 5 shows the spread for Argentina and the model-implied spread computed with the restricted estimates. As shown in the figure, the spread increases substantially with default, which leads to the much higher volatility of the spread in the sample that includes the default periods. The figure also shows that the model-implied spread moves very close to its data counterpart in the period when the two series overlap.

[Figure 5]

With the unrestricted estimates, the match between the model and the data deteriorates. The high estimate of the risk aversion coefficient leads to the consumption volatility being lower than the data. Additionally, the volatility of the spread increases substantially, and the trade balance becomes procyclical with this set of estimates. The increase in the volatility of the spread can be explained by the lower value of the reentry probability. The lower reentry probability leads to the spread remaining high for a much longer period after the default.

The model produces a countercyclical spread with both set of estimates only when all periods are considered, which shows that the increase in spread coinciding with the decline in output during the default periods is important for the model to match this statistic. Arellano (2008) obtains a countercyclical spread even though she uses only the repayment periods in her computation. The difference between her results and ours comes from the persistence parameter in the AR(1) process for output. The persistence of output in the longer sample we use is much lower than the value she uses. With a lower persistence, the model gives a procyclical spread when only the repayment periods are considered. A lower persistence makes the default incentive and the spread less sensitive to a decline in output since it is more likely that the output will recover in the next period, which reduces the negative correlation of output and spread.

Overall, the moments obtained from the model show that the parameter estimates that give the best fit between the data and the model in terms of default and output patterns fail to match the business cycle statistics observed in the data in terms of consumption, trade balance and spread. The restricted set of estimates, which are closer to the calibrated values commonly used in the literature, improve the model predictions in terms of business cycle statistics.

5 Does the Model Help Predict Default Events?

The literature on early warning systems that has developed since Kaminsky, Lizondo, and Reinhart (1998) uses model-free methods to predict different types of crises, including debt crises. These papers typically use a probit or logit estimation that regresses a crisis indicator on variables that measure the solvency and liquidity of the country as well as the strength of its macroeconomic fundamentals, such as the ratio of current account to GDP, the ratio of short-term debt to reserves, exchange rate overvaluation, reserve losses, real GDP growth rate, export growth and yield spread (see Catao and Sutton, 2002, Manasse, Roubini, and Schimmelpfennig, 2003, Berg, Borensztein, and Pattillo, 2005, Bussiere and Fratzscher, 2006, Candelon, Demitrescu, and Hurlin, 2012, and Kaminsky and Vega-Garcia, 2016, among others). In this section, we examine whether the model-implied default probability is a useful additional regressor in the reduced-form logit models (Section 5.1). We then compute pseudo out-of-sample forecasts and compare them with those based on a comparable logit model (Section 5.2).

5.1 Logit Estimation

We add the ex-ante default probability (the blue dashed line in Figure 4) as an additional regressor in the logit model that regresses the observed default variable (d^o) on a constant and other control variables commonly used in the literature. Table 4 shows that the corresponding coefficient is statistically significantly positive in all specifications, indicating that it has additional predictive power for default events.[12] The other variables used in the literature for predicting crises in general remain mostly insignificant, suggesting that they are not very useful at predicting sovereign default events.

[Table 4]

5.2 Pseudo Out-of-Sample Forecasting: A comparison with logit models

To formally compare the forecasting performance of our model with a logit-based estimation, we compute the default probability for the *observed* default events, based on our assumption that there is measurement error in the repayment regime variable. Formally, we are interested in computing the following probability:

[12] To check reverse causality, we regress the current output on its lag and the current and lagged observed default variables for on the whole sample period (from 1950 to 2010). The coefficient for the current observed default variable (d_t) turned out to be not statistically significant from zero.

$$\Pr(d_t^o | D_{t-1}^o, Y_t), \tag{7}$$

where $D_{t-1}^o = \{d_{t-1}^o, \ldots, d_0^o\}$ and $Y_t = \{y_t, \ldots, y_0\}$. This is the probability that the observed default variable in period t (d_t^o) takes a particular value given its past values (D_{t-1}^o) and the output data information up to that period (Y_t). In Appendix C, we show that this probability can be rewritten as a function of the measurement error and model-implied components using a part of the likelihood function derivation. We estimate this probability using the baseline model as well as the logit model that regresses d_t^o on a constant, d_{t-1}^o and y_t. We then use it to compute our default decision forecasts.

Figure 6 plots pseudo out-of-sample default decision forecasts under the baseline model (black solid line) and the logit model (black dashed line).[13] For period *t*, we re-estimate model parameters using output and default data from period *t*+1-40 to period *t*. Using the updated parameter values, we then compute out-of-sample default decision forecasts for each year. The figure demonstrates that the baseline model can predict the timing of the onset of the 2001 default better than the logit model. Further, the baseline model forecasts do not fluctuate under the repayment regime, though the level of the forecasts is higher than the logit-based forecasts. This is because in the baseline model, repayment decisions are projected in a stable manner due to the endogenous debt ceiling, whereas in the logit model, there is a monotonic relationship between output and default decisions. However, neither model forecasts the default duration well. Specifically, they fail to predict the timing of return to the repayment regime in 2006. Going forward, explicit modeling of default duration via debt negotiation may be useful in addressing this problem.

[Figure 6]

[13] The logit and baseline models have similar fit to the data with the corresponding log likelihood value being -16 for the logit model and -19 for the baseline model with unrestricted parameter estimates. The value implied by the baseline model with the restricted parameter estimates, however, is significantly lower (-29) because of the poor performance of the model under the restricted estimation during the default years and relatively low value of a_0 (0.72).

Lastly, we have also computed out-of-sample forecasts conditioned on the previous period's output. We find that neither model predicts the onset of default events or the timing of getting out of default when conditioned on the previous period's output.

6 Conclusion

By formally estimating the Arellano (2008) model, we find that the canonical sovereign default model is a useful indicator for Argentine default decisions. Despite the use of only output and default data in our estimation, the model accounts for overall default patterns of Argentina. The estimated sovereign default model also performs better than a comparable logit model that has been commonly used in the literature for predicting crises.

An important caveat on the model-implied business cycle properties is that if we use the unrestricted parameter estimates for model simulation, the moments from the model in terms of consumption, trade balance and interest rate spread cannot match the data. Going forward, following the developments in the theoretical literature, incorporating long-term debt and a more explicit modeling of default duration via debt negotiation and restructuring may help improve the estimated model performance in accounting for business cycle properties.

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Table 1. Summary statistics.

	y (output)	s (regime)
Repayment regime (1950, 1952-1955, 1966-1981, 1994-2000, 2006-2010)		
mean	1.02	0
std. dev.	0.04	0
min	1.11	0
max	0.96	0
Default regime (1951, 1956-1965, 1982-1993, 2001-2005)		
mean	0.98	1
std. dev.	0.05	0
min	1.05	1
max	0.84	1

Table 2. Estimated parameters.

	Baseline model (unrestricted)	Baseline model (restricted)	Arellano (2008) (annualized)
σ (risk aversion)	8.0	2	2
	(0.02)		
β (discount factor)	0.58	0.80	0.82
	(0.04)		
$1+r$ (risk-free rate)	1.03	1.03	1.07
B_0 (initial asset level)	0.00	0.00	—
$\bar{y}$ (output cost)	0.99	0.99	0.97
	(0.48)	(0.10)	
λ (reentry probability)	0.12	0.49	0.73
	(0.05)	(0.03)	
ρ	0.55	0.56	0.85
	(0.13)	(0.10)	
η	0.04	0.04	0.04
	(0.01)	(0.002)	
a_1	0	0	—
a_0	0.90	0.72	—
	(0.02)	(0.02)	

In annualized values. The numbers in parentheses are standard errors. We pre-fix the risk-free rate, and we do not report the standard error of a_1 and the initial asset level since they are estimated at the lower boundary of zero.

Table 3. Business cycle statistics.

	Data	Model (unrestricted estimates)		Model (restricted estimates)		Arellano's (2008) quarterly stat.
	All periods	Repayment periods	All periods	Repayment periods	All periods	
$\sigma(c)/\sigma(y)$	1.19	0.97	0.93	1.09	1.02	1.10
$\sigma(nx/y)$	2.58	1.86	1.75	0.72	0.82	1.50
$\sigma(spread)$	12.30	275.11	357.05	12.22	37.45	6.36
corr(c,y)	0.90	0.78	0.92	0.98	0.98	0.97
corr$(nx/y,y)$	-0.81	0.20	0.33	-0.34	-0.04	-0.25
corr$(spread,y)$	-0.81	0.03	-0.29	0.12	-0.33	-0.29

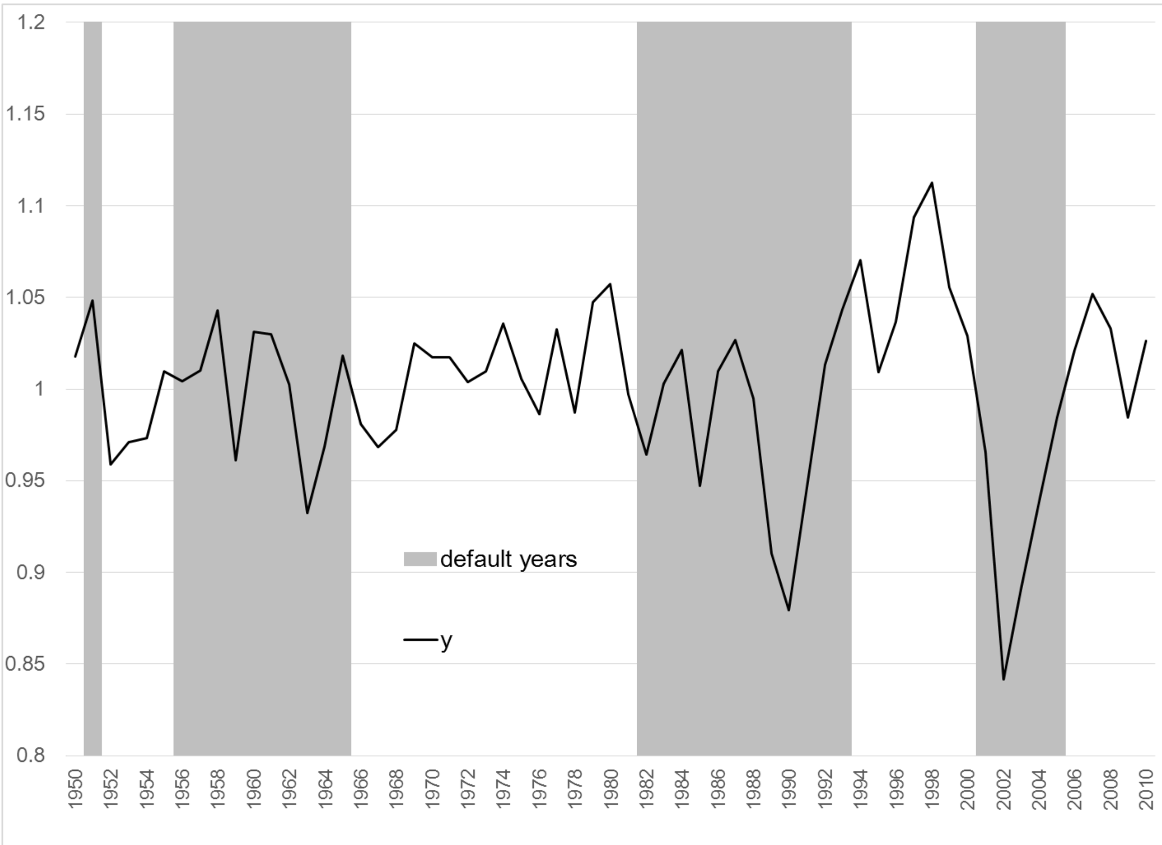
Net exports are exports minus imports; the spread is in percentages. All series except net exports and the spread are in logs. All series have been HP filtered. Standard deviations are reported as percentages. All statistics are based on annual data. Sample periods are 1950-2010 for output and consumption, 1960-2010 for net exports, and 1983-2010 for the spread.

Table 4. Logit model estimation with ex-ante default probability.

Variable	(1)	(2)	(3)	(4)
Intercept	-3.58***	-4.279***	-7.004**	-13.329*
	[-3.69]	[-3.28]	[-2.25]	[-1.75]
Delta	20.51***	22.33***	34.19**	40.22*
	[3.76]	[3.35]	[2.07]	[1.68]
RGDP growth		6.51	16.11	
		[0.74]	[0.79]	
Export growth		0.27	-11.14	
		[0.06]	[-0.93]	
Reserve growth		-0.63	0.62	
		[-0.71]	[0.17]	
Credit growth		-0.82	-7.4	
		[-0.37]	[-1.10]	
Short-term debt/Reserves			4.28*	
			[1.88]	
Debt service/Exports			-6.78	
			[-0.91]	
Spread				71.52
				[1.49]
Sample size	58	50	35	28
McFadden R^2	0.42	0.45	0.65	0.81

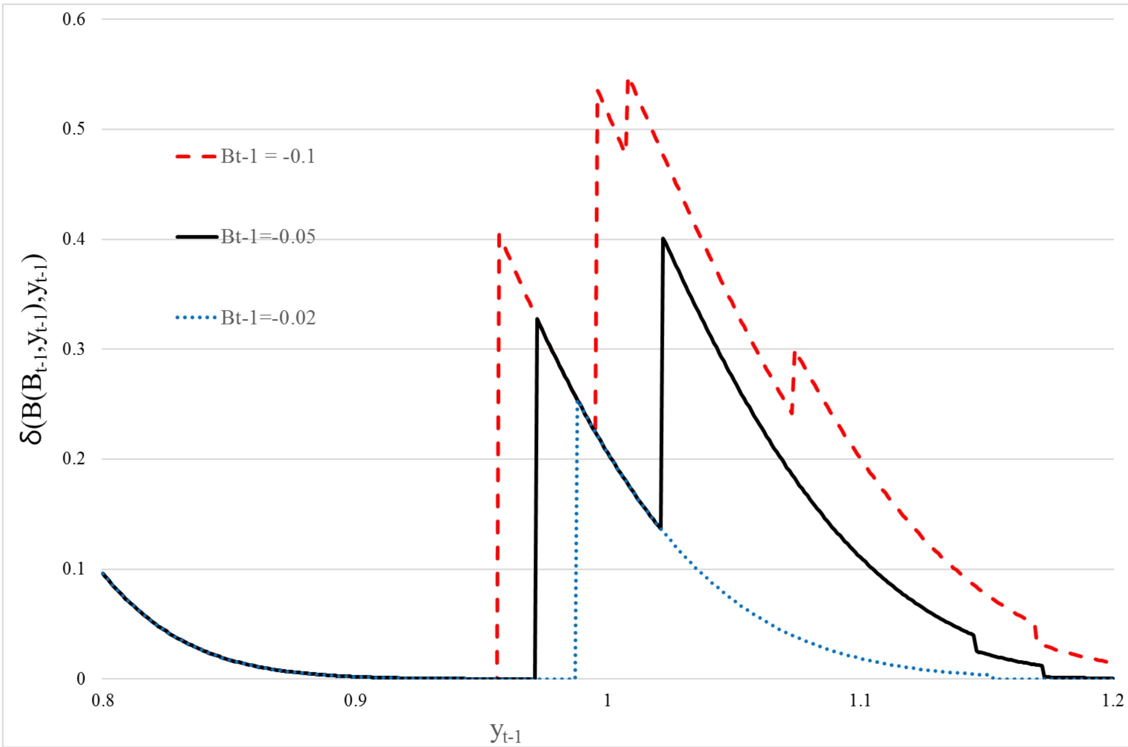
This table reports the estimated results of logit models that regresses the observed default variable on a constant, the model-implied ex-ante default probability simulated with restricted parameter estimates (delta), and other control variables. Data on exports of goods and services at constant 2005 US$ (NE.EXP.GNFS.KD), the ratio of short-term debt to reserves (DT.DOD.DSTC.IR.ZS) and the ratio of debt service to exports (DT.TDS.DECT.EX.ZS) are taken from the World Bank's International Debt Statistics. Data on total reserves in current US$ (FI.RES.TOTL.CD) and domestic credit to the private sector over GDP (FS.AST.PRVT.GD.ZS) are taken from the International Monetary Fund's International Financial Statistics. The spread data is the one shown in Figure 5. The growth variables are the log differences. The other variables are in decimals. Numbers in brackets are *t*-values. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Figure 1. Output and default data series



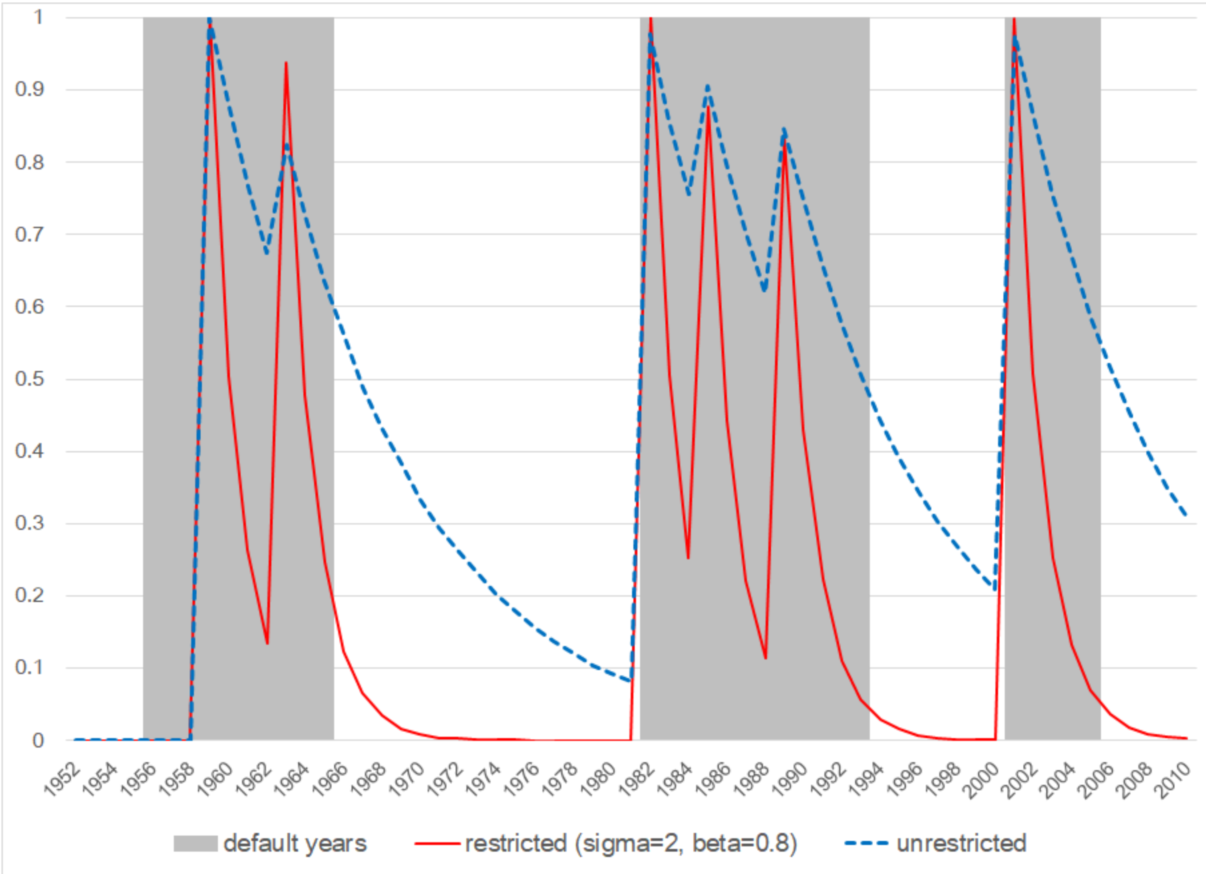
The solid line is the detrended output series for Argentina from 1950 to 2010. The shaded areas are default years.

Figure 2. Default probability function



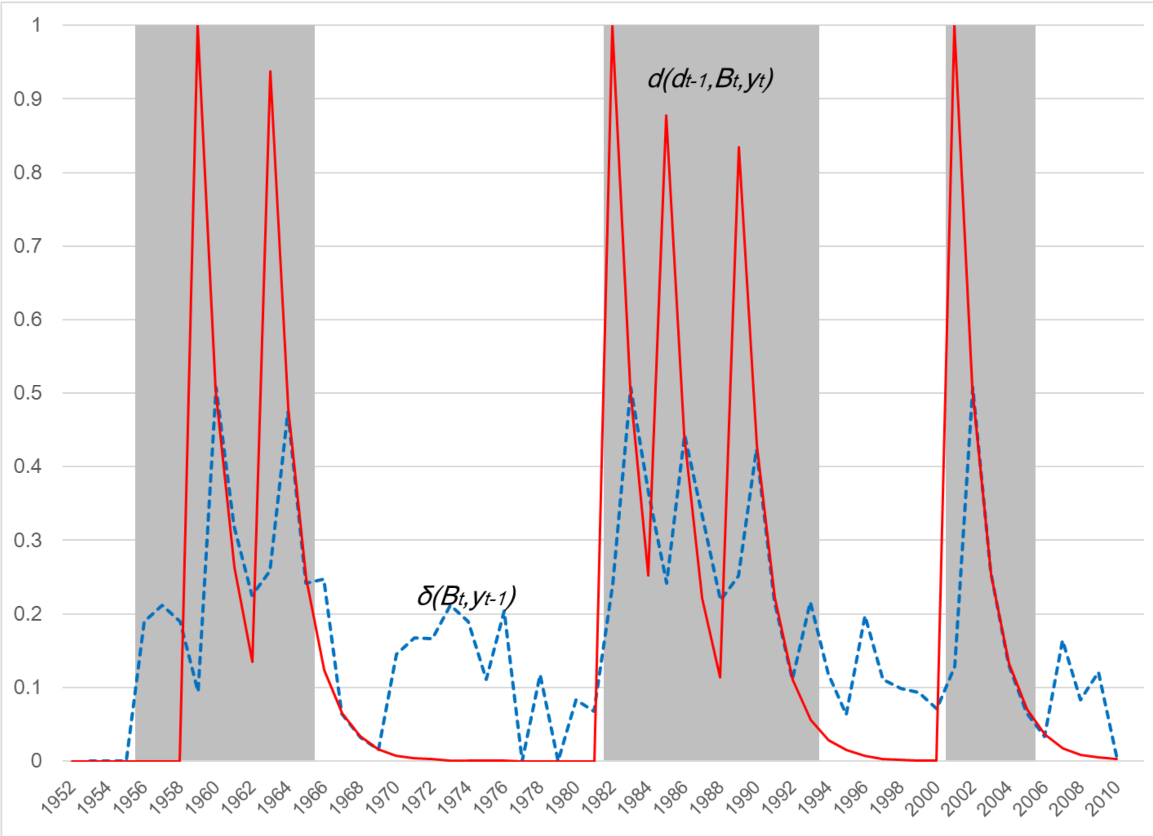
This figure plots the numerical default probability (delta) function against lagged output.

Figure 3. Simulated ex-post default probabilities



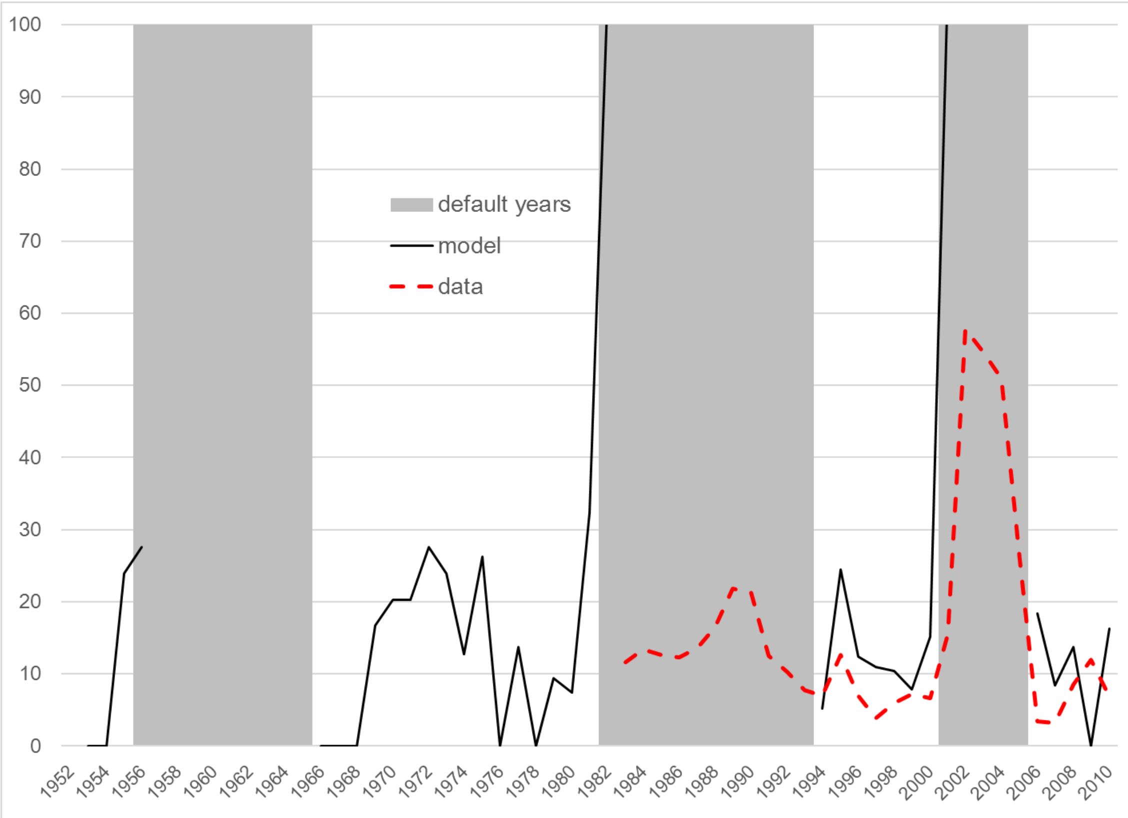
This figure plots the averages of 10,000 simulated ex-post default probabilities given the output data series and either the restricted or unrestricted parameter estimates. The red solid line and the blue dashed line are the simulated default decisions with the restricted and the unrestricted estimates, respectively. In the figure, the initial default decision is set equal to the observed default variable. The shaded areas are default years.

Figure 4. Simulated ex-ante and ex-post default probabilities



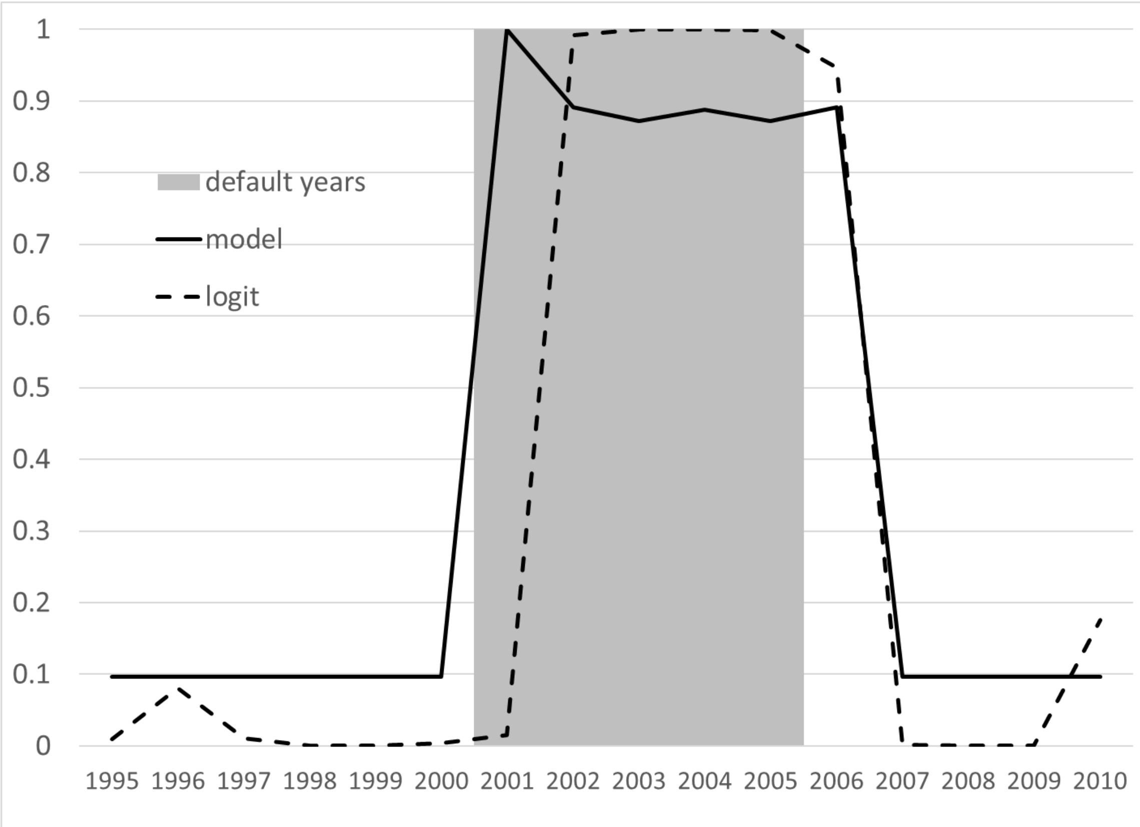
This figure plots the averages of 10,000 simulated ex-ante (blue dashed line) and ex-post (red solid line) default probabilities given the output data series and the restricted parameter estimates. In the figure, the initial default state is set equal to the observed default variable. The shaded areas are default years.

Figure 5. Simulated spread



In annualized rate in percent. The solid line shows the averages of 10,000 simulations of the interest rate spread, computed as *(1/q)-(1+r),* for the observed repayment years and the first years of default years given the output data series and the restricted parameter estimates. The red dashed line shows the spread data for Argentina. The spread data between 1983-1993 are taken from the dataset by Neumeyer and Perri (2005). The data for 1994-2010 are from EMBI Global Argentina (stripped spread). The shaded areas are default years.

Figure 6. Pseudo out-of-sample forecasts



This figure plots pseudo out-of-sample default decision forecasts using the baseline model and the logit model. Each year's default decision forecasts are computed conditional on the current year's output and the previous year's observed default decision.

Appendix A: The model

Arellano (2008)

This appendix summarizes the Arellano (2008) model. There are two regimes (d_t): default regime ($d_t = 1$) and repayment regime ($d_t = 0$). The model is set up as a planner's problem with the resource constraint given by

$$\begin{aligned} c_t &= y_t - q(B_{t+1}, y_t)B_{t+1} + B_t, && \text{under repayment,} \\ c_t &= h(y_t), && \text{under default,} \end{aligned}$$

where y is output and $h(y_t) = \overline{y}$ if $y_t > \overline{y}$ and $h(y_t) = y_t$ if $y_t \leq \overline{y}$.[14] c is consumption and q is the price of the asset. The log of output is assumed to follow the AR(1) process, i.e.,

$$\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t,\ \varepsilon_t \sim N(0, \eta). \tag{8}$$

Denoting period $t + 1$ variables with prime and period t variables with no time subscript, the value functions are given by

$$\begin{aligned} V^D(y) &= u\big(h(y)\big) + \beta E[\lambda V^R(0, y') + (1 - \lambda)V^D(y')], \\ V^R(B, y) &= \max_{B'} u(y - q(B', y)B' + B) + \beta E[\max\{V^D(y'), V^R(B', y')\}], \\ &= u(y - q(B(B, y), y)B(B, y) + B) + \beta E[\max\{V^D(y'), V^R(B(B, y), y')\}], \end{aligned}$$

where $B(.,.)$ is the savings policy function, λ is the reentry probability, and $\ln(y') = \rho \ln(y) + \varepsilon'$..

With risk-neutral lenders, the bond price satisfies

$$q(B(B, y), y) = \frac{1 - \delta(B(B, y), y)}{1 + r}, \tag{9}$$

[14] Aguiar and Gopinath (2006) point out that the sovereign debt model of Arellano (2008) cannot match the countercyclicality of interest rates, the positive correlation of interest rates, and the trade balance without an asymmetric output cost for a country in default. Without such a cost, the probability of default, the volatilities of interest rate and trade balance, and the maximum spread that the model generates decrease considerably.

where δ is the endogenous default probability given by

$$\delta(B(B,y),y) = \Pr\big(y' \in I(B')\big),$$

with $I(B) = \{y \in \mathcal{Y}: V^D(\mathrm{y}) > V^R(B,y)\}$.

Appendix B: The Likelihood Function

This appendix derives the likelihood function for the baseline model. Data on output and default variables are used in the estimation, allowing measurement error on the observed default variables.

The likelihood function for Arellano (2008)

The likelihood function of the data is given by

$$\mathcal{L} = p(d_1^o, \ldots, d_T^o, y_1, \ldots, y_T | d_0^o, y_0),$$

where the superscript o indicates *observed* default variable. $\mathcal{L}$ can be rewritten as

$$\mathcal{L} = \underbrace{p\big(\widetilde{D}_T^o \,| d_0^o, Y_t\big)}_{\mathcal{L}_A} \underbrace{p\big(\widetilde{Y}_T \,| d_0^o, y_0\big)}_{\mathcal{L}_B}$$

where $Y_t \equiv \{y_t, \ldots, y_0\}$, $\widetilde{Y}_t \equiv \{y_t, \ldots, y_1\}$, and $\widetilde{D}_t^o \equiv \{d_t^o, \ldots, d_1^o\}$.

$\mathcal{L}_B$ can be rewritten as

$$\begin{aligned} p\big(\widetilde{Y}_T \,| d_0^o, y_0\big) &= \prod_{i=1}^{T} f\,(y_t | d_0^o, Y_{t-1}), \text{(by seq. factorization)} \\ &= \prod_{i=1}^{T} f\,(y_t | y_{t-1}), \text{(the log of } y \text{ follows the AR(1))} \\ &= \prod_{i=1}^{T} \phi\left(\frac{\ln y_t - \rho \ln y_{t-1}}{\eta}\right), \end{aligned}$$

where $\phi(.)$ is the pdf of the standard normal distribution.

$\mathcal{L}_A$ can be rewritten as

$$
\begin{aligned}
p(\widetilde{D}_T^o\,|d_0^o,Y_T) &= \sum_{(d_T,\ldots,d_0)} p\left(\widetilde{D}_T^o\,, D_T|d_0^o,Y_T\right), \text{where } D_t \equiv \{d_t,\ldots,d_0\}, \\
&= \sum_{(d_T,\ldots,d_0)} p\left(d_T^o, \widetilde{D}_{T-1}^o\,, D_T|d_0^o,Y_T\right), \\
&= \sum_{(d_T,\ldots,d_0)} p\left(d_T^o|\,\widetilde{D}_{T-1}^o\,, D_T, d_0^o,Y_T\right) p\left(\widetilde{D}_{T-1}^o\,, D_T|d_0^o,Y_T\right), \\
&= \sum_{(d_T,\ldots,d_0)} p\,(d_T^o|d_T) p\left(\widetilde{D}_{T-1}^o\,, D_T|d_0^o,Y_T\right), (\text{meas. error asm.}) \\
&= \sum_{(d_T,\ldots,d_0)} p\,(d_T^o|d_T) p\left(d_{T-1}^o|\,\widetilde{D}_{T-2}^o\,, D_T, d_0^o,Y_T\right) p\left(\widetilde{D}_{T-2}^o\,, D_T|d_0^o,Y_T\right), \\
&= \sum_{(d_T,\ldots,d_0)} p\,(d_T^o|d_T) p(d_{T-1}^o|d_{T-1}) p\left(\widetilde{D}_{T-2}^o\,, D_T|d_0^o,Y_T\right), \\
&= \sum_{(d_T,\ldots,d_0)} \left[\prod_{i=1}^{T} p\,(d_i^o|d_i)\right] p(D_T|d_0^o,Y_T).
\end{aligned}
\tag{10}
$$

By the model, $p(D_T|d_0^o,Y_T)$ can be further rewritten as

$$
\begin{aligned}
p(D_T|d_0^o,Y_T) &= A\prod_{i=1}^{T} \Pr\,(d_i|D_{i-1},d_0^o,Y_T), \\
&= A\prod_{i=1}^{T} \Pr\,(d_i|d_{i-1},B_i,y_i;B_0), \\
&= A\prod_{i=1}^{T} \Pr\,(d_i|d_{i-1},B(B_{i-1},y_i),y_i;B_0),
\end{aligned}
\tag{11}
$$

where $A \equiv \Pr(d_0|d_0^o,Y_T)$. $\Pr(d_i|d_{i-1},B_i,y_i;B_0)$ in the second equality corresponds to the model-implied default decision rule which can be expressed as

$$
\begin{aligned}
\Pr(d_t = d(d_{t-1},B_t,y_t)|d_{t-1} = 0, B_t, y_t) &= 1, \\
\Pr(d_t = 1|d_{t-1} = 1, B_t, y_t) &= 1-\lambda, \\
\Pr(d_t = 0|d_{t-1} = 1, B_t, y_t) &= \lambda,
\end{aligned}
$$

where uncertainty arises only through the exogenous reentry probability λ if the country defaulted in the previous period. The last equality holds by the saving policy function. The constant, A in eq. (11) can be further rewritten as

$$\begin{aligned}\Pr(d_0|d_0^o,Y_t) &= \frac{\Pr(d_0^o|d_0,Y_t)\Pr(d_0)}{\Pr(d_0^o)}, \text{(by the Bayes rule)}\\ &= \Pr(d_0^o|d_0=1)\Pr(d_0=1)+\Pr(d_0^o|d_0=0)\Pr(d_0=0).\end{aligned}$$

By eqs. (8) and (10), the log likelihood function is given by

$$\begin{aligned}L &= \ln\left[\sum_{(d_T,\dots,d_0)} A\left\{\prod_{i=1}^{T} p\,(d_i^o|d_i)\right\}\prod_{i=1}^{T}\Pr\,(d_i|d_{i-1},B(B_{i-1},y_i),y_i;B_0)\right]\\ &+\sum_{t=1}^{T}\ln\left[\phi\left(\frac{\ln y_t-\rho\ln y_{t-1}}{\eta}\right)\right],\end{aligned}$$

where the parameter vector includes $\sigma, r, \beta, \lambda, \rho, \eta, \overline{y}, B_0, a^D, a^R$.

Appendix C: Estimated Default Probability

This appendix shows that the estimated default probability discussed in the text (eq. (5)) can be rewritten as a function of the model-implied and measurement-error components.

Ex-post

$$\begin{aligned}\Pr(d_t^o|D_{t-1}^o, Y_t) &= \frac{\Pr\left(\widetilde{D}_t^o\,|d_0^o, Y_t\right)}{\Pr\left(\widetilde{D}_{t-1}^o\,|d_0^o, Y_t\right)},\\ &\approx \frac{\sum_{(d_t,\dots,d_0)}[\prod_{i=1}^{t} p\,(d_i^o|d_i)]\,p(D_t|d_0^o, Y_t)}{\sum_{(d_{t-1},\dots,d_0)}[\prod_{i=1}^{t-1} p\,(d_i^o|d_i)]\,p(D_{t-1}|d_0^o, Y_t)}.\end{aligned}$$

Appendix D: Numerical Maximization

The solution algorithm for the baseline model is as follows:

1. Start with an initial guess for the bond price function $q(B', y)$ that corresponds to a default probability of zero for each point in the state space.

2. Using this initial price and initial guesses for $V^R(B, y)$ and $V^D(B, y)$, iterate on the Bellman equations to solve for the optimal value and policy functions.

3. Given the optimal default decision, update the price of bonds using eq. (9)). Repeat steps 2 and 3 until the bond price converges, i.e., until $\left|q^{i+1} - q^i\right| < \varepsilon$, where i represents the iteration number and ε is a very small number.

There are only ten parameters $(\sigma, r, \beta, \lambda, \rho, \eta, \overline{y}, B_0, a_0, a_1)$ in the baseline model, and many of these parameter values have restrictions on their ranges. For example, the ranges of β, λ, $\overline{y}$, a_0, and a_1 are between 0 and 1. The values of ρ and η should not be very different from the OLS estimates of the y equation alone. These restrictions enable us to compute likelihood values of all possible combinations of parameter values with reasonably fine grids. Specifically, for unrestricted parameter estimation, we set the upper and lower bounds of y-grid at 1.2 and 0.7 respectively with the grid width of 0.0025, and the upper and lower bounds of B-grid at 0 and

-0.5 with the grid width of 0.002. The parameter grid widths are set at 0.2 (σ), 0.02 (β), 0.01 for λ, λ, a_0, a_1. Once we obtain the parameter set that maximizes the value of simulated likelihood function with the above grid setting (i.e., σ=8, β=0.58, λ=0.12, $\overline{y}$=0.99, ρ=0.55, η=0.04, a_1=0, and a_0=0.90), we examine values around the estimated parameters to compute the score function. For restricted parameter estimation, we fix the values of σ and β at 2 and 0.8 respectively.