Lack of debt restructuring and lenders' credibility

(A theory of nonperforming loans)

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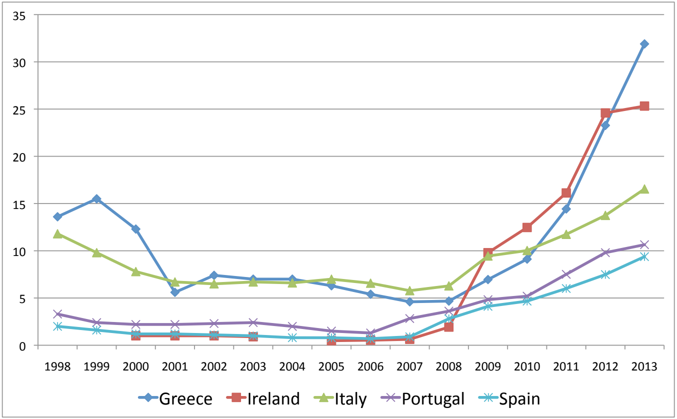
Financial crisis and nonperforming loans

- In the aftermath of a financial crisis, we observe
  - increase in uncertainty
  - persistent stagnation
- This paper:
  Accumulation of too much debt or nonperforming loans may cause these distortions

Non performing loans in some European countries



Notes: Fraction of non-performing loans in total gross loans. Source: World Bank.

Nonperforming loans

- Loans are classified as nonperforming if payments of interest and/or principals are past-due by 90 days or more (IMF).
  - They remain classified as such until written off or payments of interest and/or principal are received.
- Theoretically,
  - the contractual value of loan, D, is a payoff-relevant state variable when small;
  - when D is large, it is no longer a payoff-relevant state variable; in this case we call D a nonperforming loan.

Uncertainty

In the aftermath of a financial crisis, we observe an increase in uncertainty

- Monetary Report to Congress (July, 2010)

  "participants cited several factors that could restrain the pace of expansion $\cdots$, including $\cdots$ persistent uncertainty on the part of households and businesses about the strength of the recovery"

- Usual interpretation: an uncertainty shock
  - Changes in σ, where the variables follow $N(\mu, \sigma)$

- Our interpretation: state variable, D, is no longer payoff-relevant:
  - can no longer make actions depend on D, whereas they could in normal times;
  - higher volatility, as intertemporal smoothing with D is infeasible (conjecture).

Persistent Stagnation

In the aftermath of a financial crisis, we observe persistent stagnation

- Usual interpretation: secular stagnation hypothesis
  - Persistent changes in financial frictions (Eggertson and Mehrotra 2014)
  - Persistent changes in productivity (Gordon 2012)
- Our interpretation: state variable, D, is no longer payoff-relevant:
  - Inefficiency in production may continue persistently.
    - There arises a "debt Laffer curve" (e.g., Krugman, 1989). The amount that the lender receives from the borrower can decrease with the book value of debt.
    - Inefficiency may appear as involuntary unemployment

This paper

Accumulation of nonperforming loans may cause persistent distortions

- Debt accumulates due to negative shocks (e.g., productivity shocks);
- contractual rigidities (exogenous frictions) make debt restructuring infeasible;
- a credibility problem on the lender side arises,
  as contractual value of debt exceeds a threshold:
  - in normal times, contractual value of debt is a payoff-relevant state variable;
  - when the loan becomes too large, it becomes no longer payoff-relevant.

An example: Small debt

- $r = 0$ and $t = 0, 1, 2, \ldots$.
- The borrower earns \$ 1 million in each period.
  - He/she chooses to default if the PDV of repayments is greater than \$ 1 million;
  - maximum of PDV of repayments: $d_{\max} = 1$ million.
- $D =$ book value of debt in period 0.

- For $D \leq d_{\max}$, there is no problem with repayments.
  - e.g., the lender can offer a repayment plan: $b_0 = \$D$ and $b_t = 0$, $t \geq 1$.

  - This is a credible repayment plan.
  - D is a payoff-relevant state variable.

An example: Large debt

- $r = 0$ and $t = 0, 1, 2, \ldots$.
- The borrower earns \$ 1 million in each period.
  - He/she chooses to default if the PDV of repayments is greater than \$ 1 million;
  - maximum of PDV of repayments: $d_{\max} = 1$ million.
- D = book value of debt in period 0.

- Suppose that $D = 2$ million $(> d_{\max})$, and D cannot be adjusted.
  - The lender could offer a repayment plan: $b_0 = 1$ and $b_t = 0$, $t \geq 1$.
  - But it is not credible, because, in period 1, $D_1 = 1$, and the lender can demand the borrower to repay another 1 million.
  - Expecting it, the borrower will choose to default in period 0.
  - D is no longer a payoff-relevant state variable.

Literature

- Model of long-term debt contract
  - with state-contingent debt: Albuquerque and Hopenhayn (2004),
  - with non state-contingent debt: our model.
- Debt overhang
  - with new and old lenders: Myers (1971),
  - with only single lender: our model.

- time periods: $t = 0, 1, \ldots$.
- productivity: $s \in \{s_L,\ s_H\}$, where $0 \leq s_L < s_H$.
- a lender (bank) and a borrower (firm).
- two types of funds provided by the lender:
    - D_0 = initial amount of long-term loan;
    - k_t = short-term loans (intra-period loans) in each period $t \geq 0$.
- β = common discount factor.
    - R = rate on short-term loans k_t.
    - $r = \beta^{-1} - 1$ = rate on long-term loans (inter-period loans).
- $F(s_t, k_t)$ = production (revenue) function of the firm.
- b_t = repayments on the long-term loans D_t in periods $t \geq 0$.

Value of the borrower

- x_t = dividends to the borrower (owner of the firm):

$$x_t = F(s_t, k_t) - Rk_t - b_t.$$

- Limited liability:

$$x_t \geq 0, \qquad \forall t \geq 0.$$

- V_t = PDV of dividends (value of the borrower):

$$V_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} x_i = x_t + \beta \mathbb{E}_t V_{t+1}.$$

Limited commitment

- The firm can choose to default in any period t, after receiving working capital k_t.
- $G(s_t, k_t) =$ the value of the outside opportunity of the firm.
- The bank would receive none when the firm defaults.
- Enforcement constraint:

$$V_t \geq G(s_t, k_t), \qquad \forall t \geq 0.$$

Banks

- Banks are competitive in the sense that they take as given
  - market rate for short-term lending: R,
  - market rate for long-term lending: $r = \beta^{-1} - 1$.
- Let d_t be the PDV of repayments of periods $t \geq 0$:

$$d_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} b_i \geq 0.$$

Contractual value of debt

- A contract specifies
  - the interest rate, $r = \beta^{-1} - 1$,
  - the initial value of debt, D_0.
- In each period t, the repayments made prior to that period are verifiable.
- Let D_t be the contractual value (book value) of debt in period t:

$$D_t = \beta^{-1}(D_{t-1} - b_{t-1}).$$

- Then D_t is verifiable and can be used as a state variable.
- Note that it is a legal commitment that the bank cannot require repayment more than D_t:

$$b_t \leq D_t$$

Contractual rigidities

- Assume that debt restructuring is not feasible due to exogenous rigidities.
  - The bank cannot reduce the contractual value of debt from D_t to $\hat{D}_t$, where $\hat{D}_t < D_t = \beta^{-1}(D_{t-1} - b_{t-1})$.
- The contractual rigidities may arise from, e.g.,
  - war of attrition due to bargaining frictions,
  - bank's preference not to trigger a bank run.

Bank's problem

- Bank maximizes PDV of repayments:

$$
\begin{aligned}
\max \quad & d_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t b_t \\
\text{s.t.} \quad & V_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} \big[F(s_i, k_i) - Rk_i - b_i \big] \geq G(s_t, k_t), \\
& F(s_t, k_t) - Rk_t - b_t \geq 0, \\
& D_t = \beta^{-1}(D_{t-1} - b_{t-1}), \\
& b_t \leq D_t.
\end{aligned}
$$

Recursive formulation

- Consider Markov equilibrium with the state variables $(s_t,\ D_t)$.
- Given expectations on borrower's value, $V^e(s, D)$, bank solves

$$
\begin{aligned}
d(s, D) = \max_{b,k,V} \quad & b + \beta \mathbb{E} d(s_{+1}, D_{+1}) \\
\text{s.t.} \quad & V = F(s, k) - Rk - b + \beta \mathbb{E} V^e(s_{+1}, D_{+1}), \\
& F(s, k) - Rk - b \geq 0, \\
& G(s, k) \leq V, \\
& D_{+1} = \beta^{-1}(D - b), \\
& b \leq D.
\end{aligned}
\tag{1}
$$

Equilibrium condition is

$$V(s, D) = V^e(s, D).$$

Contractual value and real value of debt

- Contractual value of debt: D.
- Real value of debt: $d(s, D) \, (= \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} b_i)$.
  - In the deterministic case, $s_H = s_L$, with small debt D:

    $$d(D) = D.$$

  - In the stochastic case, $s_H > s_L$:

    $$d(s, D) \leq D.$$

Characterization of equilibrium

- $k^*(s)$ = first-best level of production:

$$k^*(s) = \arg\max F(s,k) - Rk.$$

- Threshold $D_{\max}(s)$:

$$D_{\max}(s) = \max_{D_{+1}(s,D) \leq D} D.$$

  - For $D > D_{\max}(s)$,

$$D_{+1}(s,D) > D.$$

Markov equilibrium

- Markov equilibrium exists, though it may not be unique. See Appendix.
  - Proof is given for a discretized version of the model.
- The equilibrium $\{k(s,D),\ b(s,D),\ d(s,D),\ V(s,D)\}$ satisfies that
  - for $D \leq D_{\max}$,
    - borrower repay debt as much as possible by setting dividend zero: $F(s,k) - Rk - b = 0$;
    - $\{k(s,D),\ V(s,D)\}$ are decreasing in D;
    - $\{D,\ k(s,D)\}$ can converge to first-best, $\{0,\ k^*(s)\}$, with positive probability;
    - D is a payoff-relevant state variable;
  - for $D > D_{\max}$,
    - $\{k(s,D),\ b(s,D),\ d(s,D),\ V(s,D)\} = \{k_{\text{npl}}(s),\ b_{\text{npl}}(s),\ d_{\text{npl}}(s),\ V_{\text{npl}}(s)\}$;
    - D can never be repaid in full;
    - D is no longer a payoff-relevant state variable.

When $D > D_{\max}$

- Suppose that, in some period ("period t_0"),

$$D_{t_0} > D_{\max}(s),$$

  as a result of a continuation of low productivity.
- Then, for any feasible path $\{b_t, k_t, V_t, d_t\}$,

$$D_{t+1} > D_t, \quad \forall t \geq t_0.$$

Proposition: For $D > D_{\max}$, the solution to (1) does not depend on D, i.e., $\{k(s,D), b(s,D), d(s,D), V(s,D)\} = \{k(s), b(s), d(s), V(s)\}$.

- Let $\{k_t, b_t, d_t, V_t\}$ be $\{k(s_t,D_t), b(s_t,D_t), d(s_t,D_t), V(s_t,D_t)\}$ with $D_0 = D$,
- $\{k_t, b_t, d_t, V_t\}$ is the solution to the sequential problem (2) with $D_0 = D$.

$$
\begin{aligned}
d_0 = \max_{k_t, b_t, V_t} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t b_t && (2) \\
\text{s.t.} \quad & V_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} \big[F(s_i,k_i) - Rk_i - b_i\big] \geq G(s_t,k_t), \\
& F(s_t,k_t) - Rk_t - b_t \geq 0, \\
& D_t = \beta^{-1}(D_{t-1} - b_{t-1}), \\
& b_t \leq D_t. \qquad (\Leftarrow \ \text{This constraint never binds.})
\end{aligned}
$$

- $\{k_t, b_t, d_t, V_t\}$ is also the solution to (2) with any $D_0 = D' > D_{\max}$.
- Thus, $\{k_t, b_t, d_t, V_t\}$ should be independent of D_0.

Markov equilibrium with $D > D_{\max}$

- As D is no longer a payoff-relevant state variable, the problem of the bank can be written as follows. Given the expectations, $V^e(s)$,

$$d(s) = \max_{k,b} \; b + \beta \mathbb{E} d(s_{+1})$$
$$\text{s.t.} \quad G(s,k) \le F(s,k) - Rk - b + \beta \mathbb{E} V^e(s_{+1}),$$

  which reduces to

$$\max_{k} \; F(s,k) - Rk - G(s,k) + \beta \mathbb{E}(V^e(s_{+1}) + d(s_{+1})).$$

- The solution is given by

$$k_{\text{npl}}(s) \equiv \arg\max F(s,k) - Rk - G(s,k).$$

Markov equilibrium with $D > D_{\max}$

- $k_{\text{npl}}(s)$ = worst level of production:

$$k_{\text{npl}}(s) = \arg\max F(s, k) - Rk - G(s, k).$$

- Note: $k_{\text{npl}}(s) < k^*(s)$.

Lemma: The solution to (1) satisfies that $k(s, D) \geq k_{\text{npl}}(s), \quad \forall D \geq 0$.

Persistence of inefficiency

- Equilibrium dynamics:

$$k(s_t) = k_{\text{npl}}(s_t) \quad < k^*(s_t),$$

$$V(s_t) = V_{\text{npl}}(s_t) \equiv G(k_{\text{npl}}(s_t)) \quad < V^*(s_t),$$

$$b(s_t) = b_{\text{npl}}(s_t) \equiv F(s_t, k_{\text{npl}}(s_t)) - Rk_{\text{npl}}(s_t) - V_{\text{npl}}(s_t) + \beta \mathbb{E}_t V_{\text{npl}}(s_{t+1}),$$

$$d(s_t) = d_{\text{npl}}(s_t) \equiv b_{\text{npl}}(s_t) + \mathbb{E}_t d_{\text{npl}}(s_{t+1}) \quad < d_{\max}(s) = \max_D \; d(s, D).$$

Persistence of inefficiency

- **Debt Laffer curve**:

$$d(s_t) = d_{\text{npl}}(s_t) \quad < d_{\max}(s) = \max_D \; d(s, D).$$

  - $d(s, D)$ is increasing in D for $D \leq \bar{D}(s) \equiv \arg\max_D d(s, D) \leq D_{\max}$.
  - $d(s, D)$ is decreasing in D for $D > \bar{D}(s)$.

- **Involuntary unemployment**
  in the general equilibrium with firms with $D > D_{\max}$:
  - supply is given: $k = 1$,
  - if $F(s, 1) - R - b_{\text{npl}}(s) + \beta \mathbb{E} G(s, 1) < G(s, 1)$ for any $R > 0$, then $k_{\text{npl}}(s) < 1$.
  - there arises an excess supply of k: $1 - k_{\text{npl}}(s)$.

Summary

- Suppose that $D_{t_0} > D_{\max}$.
- If debt restructuring is feasible,
  - the PDV of repayments, $d(s, D)$, can be maximized by rewriting the contract and reducing the amount of debt to

$$\bar{D}(s) \equiv \arg\max_{D} d(s, D) \leq D_{\max}.$$

- Without debt restructuring,
  - contractual value of debt, D, is no longer a payoff-relevant state variable;
  - equilibrium path cannot be contingent on D;
  - in this case, inefficiency will continue forever.

Extensions

- Numerical experiments to compare the volatilities of macro variables:

Conjecture: Volatility is larger for $D > D_{\text{max}}$ than for $D \leq D_{\text{max}}$.

  - When $D > D_{\text{max}}$, the inter-temporal smoothing cannot be implemented as D cannot be used as a state variable;
  - thus, an increase of debt from D to D', where $D < D_{\text{max}} < D'$, can be regarded as the source of uncertainty shock.

- Further extensions on contractual rigidities are
  - explicit cost of adjusting D_t.
  - bargaining.